MEASUREMENT OF TRANSMISSION LINES WITH VNA

Let's start by defining what is meant by a transmission line. A transmission line is a device used to transfer electrical energy from one point to another in space, with minimal losses. Usually, it is a confined space (with a length greater than $\lambda/20$) where electric charges move or electromagnetic waves are guided. The most commonly used types are:

- Twin-wire conductors;
- Coaxial conductors;
- Waveguides.

Each transmission line is characterized by a cutoff frequency that identifies the minimum operating frequency. However, there is not always a maximum operating frequency. Typically, the maximum frequency of use is limited by excessive losses or the appearance of other modes of propagation, as is the case with waveguides. In our specific interest, the twin-wire and coaxial lines, the mode of propagation is TEM (Transverse Electromagnetic). This means that the electric field and the magnetic field generated by the currents are always perpendicular to each other and both are perpendicular to the direction of propagation. Figure 1 shows an example of an EM wave propagating in a vacuum at the speed of light.

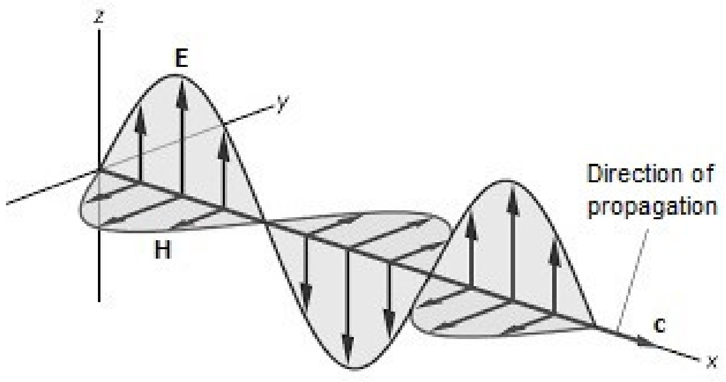

Figure 1: TEM propagation mode in free space.

In figure 2 is represented the TEM wave propagating in a coaxial line

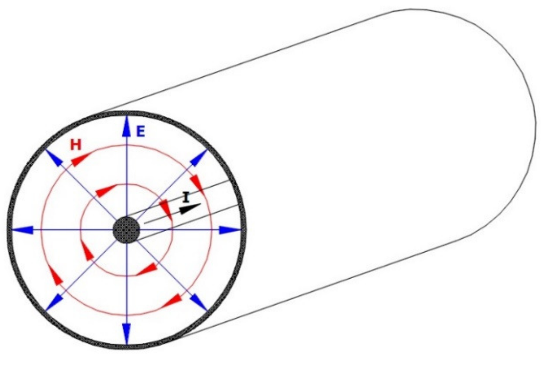

Figure 2: TEM propagation mode in coaxial lines.

In figure 3, the distribution of EM fields in a twin-wire line is represented.

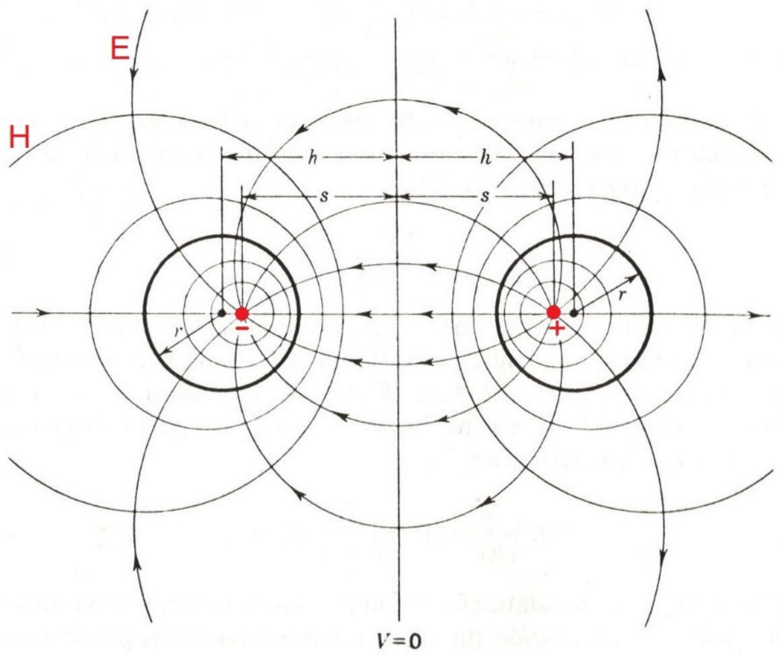

Figure 2: TEM propagation mode in twin-wire lines.

In all cases, the EM wave propagates in the dielectric.

In dielectrics other than vacuum, EM waves travel at lower speeds than the speed of light, which depends on the relative dielectric constant ε_r and the relative magnetic permeability μ_r of the dielectric in which they propagate. The propagation velocity v_p is calculated using the following equation:

$$v_p = \frac{c}{\sqrt{\epsilon_r \mu_r}} \cong \frac{c}{\sqrt{\epsilon_r}} \qquad 1.1$$

As you can see, the propagation velocity of the EM wave depends solely on the dielectric constant ε_r because the relative magnetic permeability of dielectrics is approximately $\mu_r \approx 1$.

The ratio between the propagation velocity v_p and the speed of light c is called the Velocity Factor (VF):

$$VF = \frac{v_p}{c} \cong \frac{1}{\sqrt{\varepsilon_r}} \qquad 1.2$$

In a transmission line, the ratio between the voltage v(x,t) across the two conductors and the current i(x,t) has a constant value in all sections of the line. This ratio has the dimensions of resistance and is called the characteristic impedance Z_0. The expression for the characteristic impedance of a line can be derived from electrical parameters (L and C) or physical parameters of the line. In fact, for a coaxial line:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\, ln\frac{b}{a} \qquad 1.4$$

where b is the diameter of the dielectric and a is the diameter of the inner conductor, both in meters. For a twin-wire line, it can be approximated as:

$$Z_0 = \sqrt{\frac{L}{C}} \approx \frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon}} ln\frac{2D}{d} \qquad 1.5$$

where D is the distance between the two centers and d is the conductor diameter, both in meters.

In both cases, L and C represent the inductance and capacitance per unit length of the line. The capacitance can be measured using a capacitance meter by measuring the capacitance between the two conductors with an open line, while the inductance can be obtained using an inductance meter by measuring the inductance between the two conductors with the line shorted. Taking the square root of the ratio between these two values provides an idea of the characteristic impedance. The characteristic impedance of a line is one of the most important parameters to know. Real transmission lines also have signal attenuation due to losses in the conductor (skin effect) and dielectric. The equation expressing the attenuation A_f per unit length in dB/m is as follows:

$$A_f = K_1\sqrt{f} + K_2 f \;\; [\mathrm{dB/m}] \qquad 1.6$$

The parameter K_1 accounts for losses in the conductor and is proportional to the square root of the frequency f, while the parameter K_2 accounts for losses in the dielectric and is proportional to the frequency.

The three parameters Z_0, VF, and A_f characterize the transmission line, and now we will see how to measure them with a VNA.

Let's start with the method to determine the characteristic impedance Z_0 using a VNA.

The first step is to properly terminate the cables with suitable connectors.

Next, the length of the line to be tested in meters must be accurately measured.

The length is measured between the respective reference planes of the connectors.

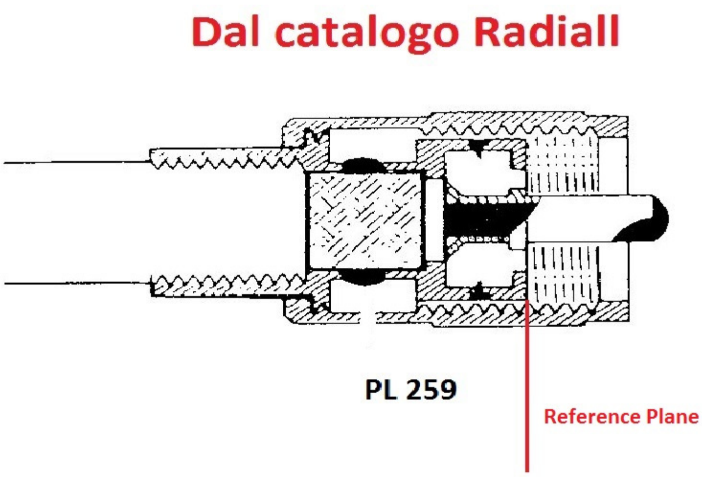

Figure 4: Datasheet of the PL 259 connector.

In the case of PL 259 connectors, the conventional reference plane is the one shown in Figure 4, and the measurement should be as accurate as possible. To determine the characteristic impedance of our unknown line we will use the lambda/4 transformation technique. The technique involves terminating the line with a non-inductive resistive load R_B of a known value but different from the hypothetical Z_0 of the line, for example, 100 ohms (Figure 5).

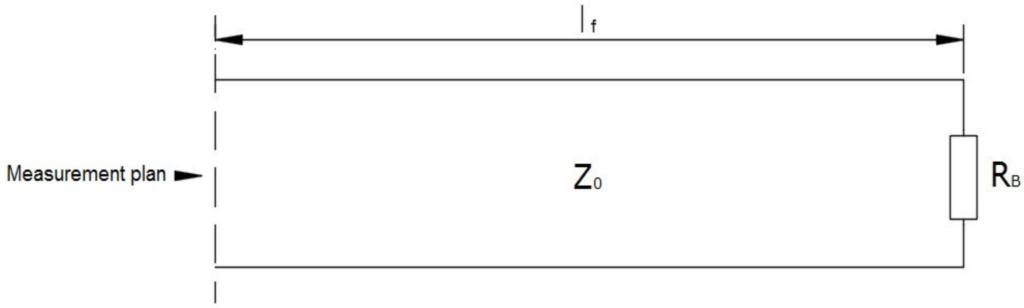

Figure 5: Sketch of the loaded line.

Afterwards, the input impedance Z_i of the line connected to port 1 is measured using the VNA. The measurement is performed within the frequency range starting from the minimum frequency f_{Min}=100kHz up to the frequency obtained from the following formula:

$$f_{MAX} = \frac{75}{l_f} \text{ MHz} \quad 1.7$$

After calibration, the image that the VNA will display on the screen will be something like the following:

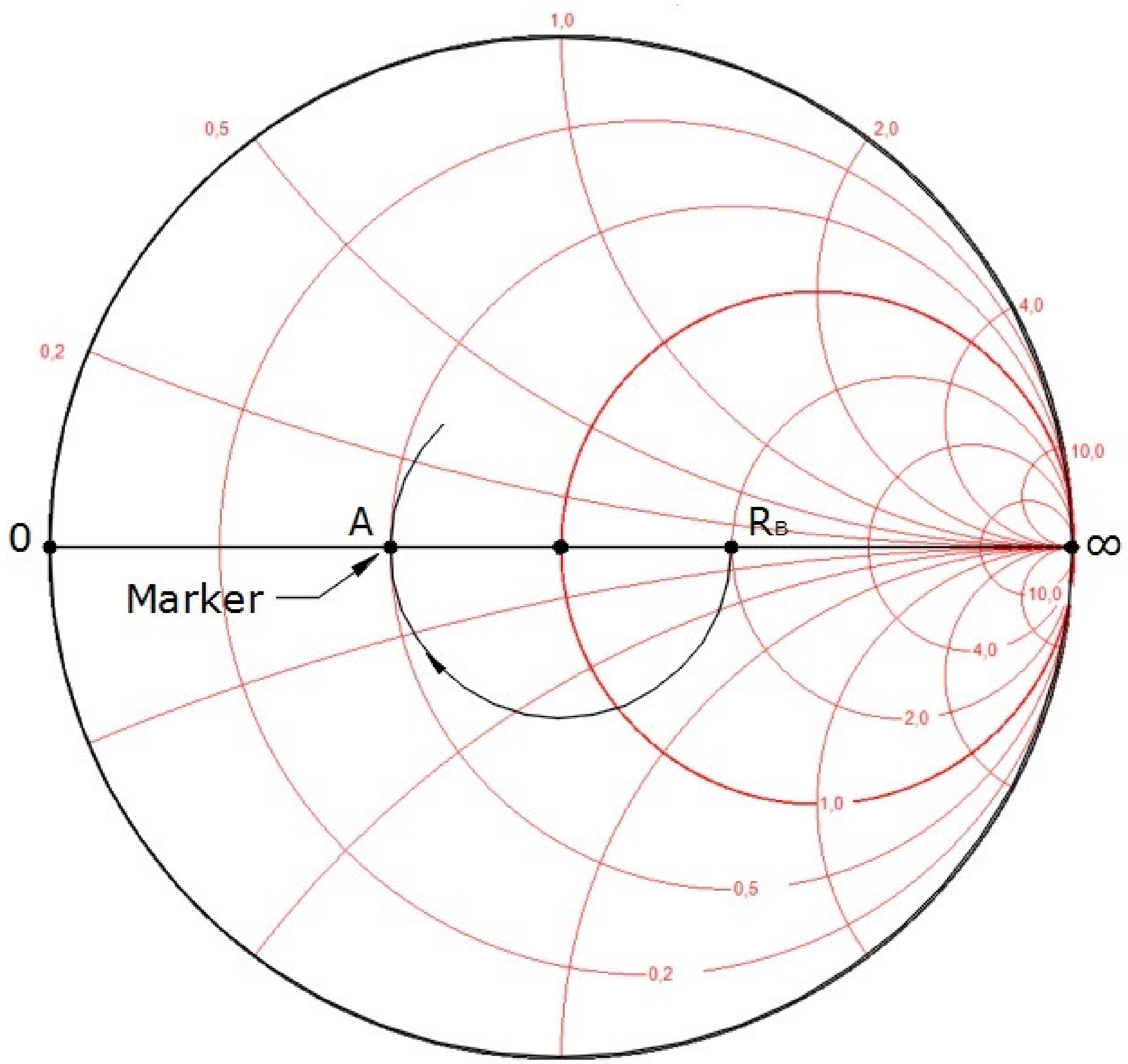

Figure 6: Impedance Z diagram on the VNA.

The line will be represented by an arc of a circle starting from the load R_B, which acts as termination, and passing through the diameter of the Smith chart at point A. Point A will be located at a distance of exactly lambda/4; a half-circle on the Smith chart. Based on the equation describing the impedance variation along a line, point A will have a pure resistance R_A, related to R_B by the following relationship:

$$Z_0 = \sqrt{R_A \cdot R_B} \quad 1.8$$

Therefore, with known resistances R_A and R_B, the characteristic impedance Z_0 of the line under test can be easily calculated from equation 1.8.

Using the VNA marker, we aim to identify, as accurately as possible, the resistance at point A (where the imaginary part should be close to zero). Besides noting the resistance at point A, we also note its frequency f_A.

The frequency at point A corresponds precisely to lambda/4 electrical length, so the electrical length le of the line can be calculated as follows:

$l_e = \frac{75}{f_A}$ con f_A in MHz. 1.9

Since we previously measured the physical length l_f of the line, we can easily determine the velocity factor VF:

$VF = \frac{l_f}{l_e}$ 1.10

The third and final parameter to measure is the linear attenuation A_f of our unknown line. To do this, we need to select the frequency of interest at which to measure the attenuation and obtain two non-dissipative loads to replace the R_B resistance: an open circuit and a short circuit.

We set the VNA center frequency to the working frequency with an appropriate span for the intended use. Calibrate the VNA and display the magnitude of the reflection coefficient in dB.

Then, at the working frequency, measure the magnitude of the reflection coefficient with the line terminated with the open circuit Γ_o and then with the short circuit Γ_s. Using the following formula, we can calculate the attenuation A_s for the entire line segment:

$A_s = \frac{\Gamma_O + \Gamma_S}{4}\ dB$ 1.11

Finally, the attenuation per 100 meters A_f is obtained by dividing A_s by the physical length l_f of the line and then multiplying by 100:

$A_f = \frac{A_s}{l_f} 100\ \ dB/100m$ 1.12