LEARNING TO USE THE VECTOR NETWORK ANALYZER (VNA) CHARACTERIZING TRANSMISSION LINES

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1. Introduction

Even transmission lines, such as twin-wire lines or coaxial lines, can be characterized in a simple and reasonably accurate way with a VNA (Vector Network Analyzer). The adjective "reasonably" is appropriate because the gold standard for these types of measurements is TDR (Time Domain Reflectometry), covered in RR n° 1 of 2012. However, we will see that good results can also be obtained with a VNA.

2. Theory

This time, the theory is quite simple. The principle we use to measure the characteristic parameters of the lines under examination is that of the quarter-wave transformer. In other words, we exploit the fact that if we traverse a transmission line from the load to the generator, rotating around the center of the Smith chart in a clockwise direction, assuming the line has no losses, we will return to the starting point Z_L after exactly half a wavelength. (See Figure 15-1).

This means that the load impedance Z_L placed at the end of our transmission line will reappear unchanged after each rotation, that is, every half wavelength ($\lambda_e/2$). Along the path, we also encounter points with real impedance $Z_A=R_A+j0$ and $Z_B=R_B+j0$ (Figure 15-1).

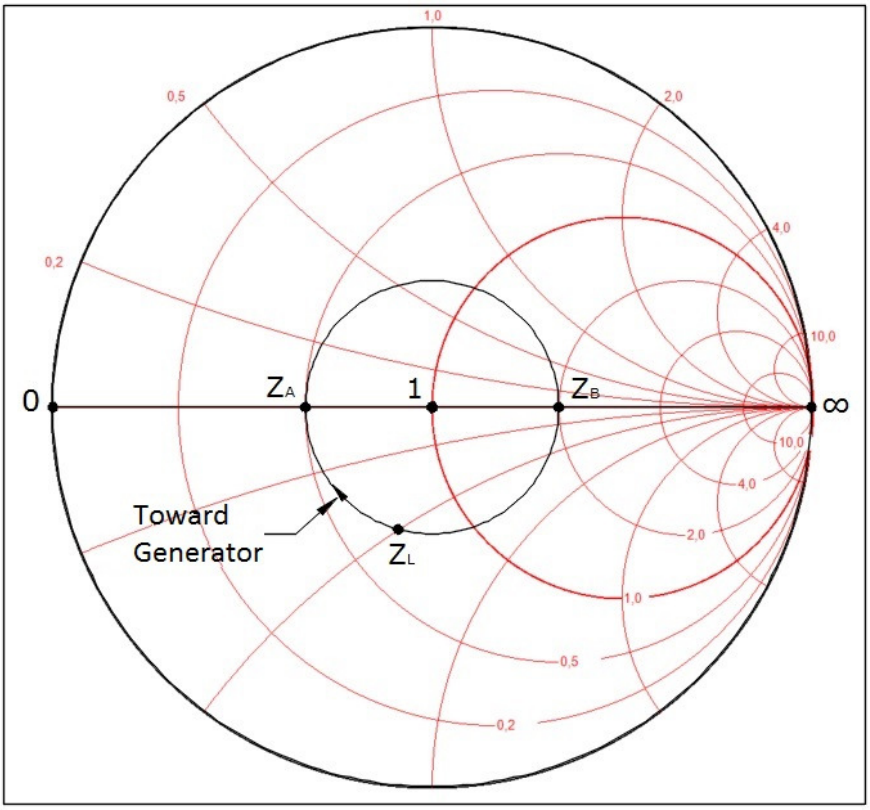

Figure 15-1: Impedance circle along a closed line on Z_L.

The points are quite important for our discussion because they lie on the real axis and are half a rotation apart, which corresponds to a quarter-wavelength electrical distance ($\lambda_e/4$) or, if you prefer, S_{11}, also known as the reflection coefficient Γ (gamma), is rotated by 180°.

Without getting too lengthy, the relationship between the two impedances Z_A and Z_B is known [1] and depends on the characteristic impedance Z_0 of the intervening line according to the following equation:

$Z_0 = \sqrt{Z_A \cdot Z_B}$ 15.1

Equation 15.1 is very simple because both Z_A and Z_B are real impedances and it coincides with the geometric mean of the real parts of Z_A and Z_B. Henceforth, we will simply refer to them as $R_A=Z_A$ and $R_B=Z_B$ to emphasize that we have chosen them on the real axis, meaning their imaginary parts are zero. Therefore, equation 15.1 becomes:

$Z_0 = \sqrt{R_A \cdot R_B}$ **15.2**

From an electrical perspective, the equivalent circuit is shown in Figure 15-2.

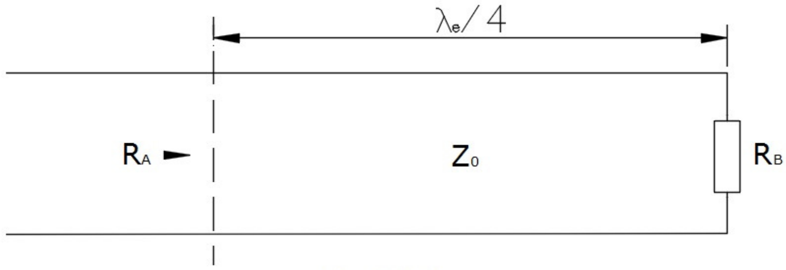

Fig.15-2

Figure 15-2: Impedance transformation after λ/4.

In Figure 15-2, the real impedance R_B is transformed into the real impedance R_A after traversing a segment of the transmission line that is a quarter-wavelength long with characteristic impedance Z_0. This is why it is called a quarter-wave transformer.

It goes without saying that if we know R_A and R_B, we can use equation 15.2 to calculate the Z_0 of the line $\lambda_e/4$ that connects them.

However, quarter-wavelength electrical (λ_{qe}) is only applicable at a specific frequency. In an ideal line, the wave propagates at the speed of light c=300.106 m/sec. Therefore, from the motion equation, we can derive the wavelength and then the frequency:

$\lambda_{qe} = \lambda_e/4 = \frac{300}{4 \cdot f_q} = \frac{75}{f_q}$ **15.3**

The result will be in meters if the quarter-wavelength frequency f_q is in MHz.

In practice, the wave in a real line does not travel at the speed of light but at a slower pace. Hence, equation 15.3 introduces the velocity factor VF, which accounts for the reduction in velocity. Thus, equation 15.3 becomes:

$\lambda_{qf} = \frac{VF \cdot 75}{f_q}$ **15.4**

The velocity factor VF is always less than one and is another characteristic parameter of transmission lines.

The last parameter necessary for complete knowledge of the line is its attenuation. To measure the attenuation, we will use a clever method described in [2]. Although it is not as accurate as the method we have already seen in section 12 [3] when we discussed measurements through two-port systems, it allows us to measure the attenuation of our segment in a simple and fast way.

The method utilizes the definition of the reflection coefficient Γ, which expresses how much the reflected wave from the load, placed at the end of the line, has been reduced compared to the wave sent by the VNA on the measurement plane. The cleverness lies in placing a non-dissipative load, such as an open or a short, at the end of the line. By doing so, the energy of the wave will be entirely reflected back towards the VNA ($|\Gamma|^2 = 1$). If, on the measurement plane of the VNA, we observe that the reflected wave has reduced, then the cause of the attenuation is certainly due to dissipation along the line during the round-trip, because we have chosen a non-dissipative load.

In theory, this is correct, but in practice, the non-dissipative load should be understood as minimally dissipative, both for an open and a short. Additionally, the line, in addition to the dissipative component, also has small non-dissipative fractions that cause disturbances.

To increase the accuracy of the actual measurement, we perform measurements with both an open line and a short line, and then calculate the geometric mean of the two values of |Γ| obtained [2]. Using a short and then an open ensures a 180° phase rotation on the reflected wave, allowing the geometric mean to reduce errors.
The magnitude of the reflection coefficient |Γ|, which is the radius of the circle in Figure 15-1, can also be expressed in terms of return loss RL, which is more convenient. In decibels, the reciprocal is indicated by a minus sign in front, so:

$$RL = 20log\,1/|\Gamma| = -20log|\Gamma|\text{dB} \qquad 15.5$$

Since the waves are measured at the start, the return wave undergoes attenuation twice due to the round trip along the line L_C. In decibels, the product of two cascaded attenuations is expressed as the sum of the attenuations. Since the attenuation L_C of the line remains the same, we have:

$$L_C = \frac{RL}{2}\text{dB} \qquad 15.6$$

The geometric mean of the RL values obtained with an open load RL_O and a short load RL_S is expressed as the square root of the product of the two values. In decibels, it becomes the arithmetic mean of the attenuations L_C. The practical formula is as follows:

$$L_C = \frac{L_{CO}+L_{CS}}{2} = \frac{RL_O+RL_S}{4}\text{dB} \qquad \mathbf{15.7}$$

Obviously, the line attenuation changes depending on the frequency [3, section 12], so we need to measure the RL at the frequency of interest.

3. From theory to practice

First, let's see how we can use the knowledge gained to determine the unknown characteristic impedance Z_0 of our line and its velocity factor VF with the help of the VNA.
As we already know [3], the VNA is capable of measuring the S_{11} parameter (also known as the reflection coefficient Γ) on port 1. From the reflection coefficient Γ, we can derive the impedance presented on the measurement plane of port 1 at various frequencies within the instrument's span.
During the sweep, there will also be a frequency that corresponds to a quarter-wavelength, highlighting the quarter-wave transformation and allowing us to use the previous formulas.
Therefore, it is important that the span includes the frequency that corresponds to the quarter-wavelength, without going too far to avoid making too many rotations around the center of the Smith chart.
To find the maximum frequency f_{MAX} where we should stop the span, we need to measure the physical length lf (in meters) of our unknown line and then utilize equation 15.3 as follows:

$$f_{MAX} = \frac{75}{l_f}\text{ MHz} \qquad 15.8$$

The frequency obtained from equation 15.8 will be the end frequency of the span, while the start frequency will be the minimum of the instrument.
Once the minimum and maximum frequencies are set, we need to calibrate port 1 of the VNA using the SOL kit on the actual measurement plane. After that, we will have the instrument ready to measure anything presented on the conventional reference plane, but we will need to move it to coincide with the point where we connect the start of our unknown line. To shift the conventional reference plane to the measurement plane, we need to add twice the delay of the connector we use to connect the line in the appropriate menu. For example, in the NanoVNA SAA-2N, it can be found under Display → Scale → EL.Delay.
Next, we need to take a resistance, which is anti-inductive within the span frequencies, with a known value that is preferably higher than the hypothetical value of Z_0 for the line under examination, and also different from the 50 Ohms of the instrument (e.g., 100 Ohms). This resistance needs to be accurately measured as it will serve as our R_B.
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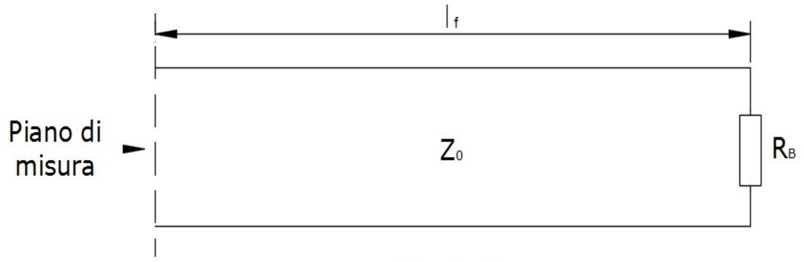

Figure 15-3: Measurement circuit with VNA.

Let's set up the VNA to display at least the Smith chart and impedance. Start the scan, and if all goes well, we will obtain the diagram shown in Figure 15-4.

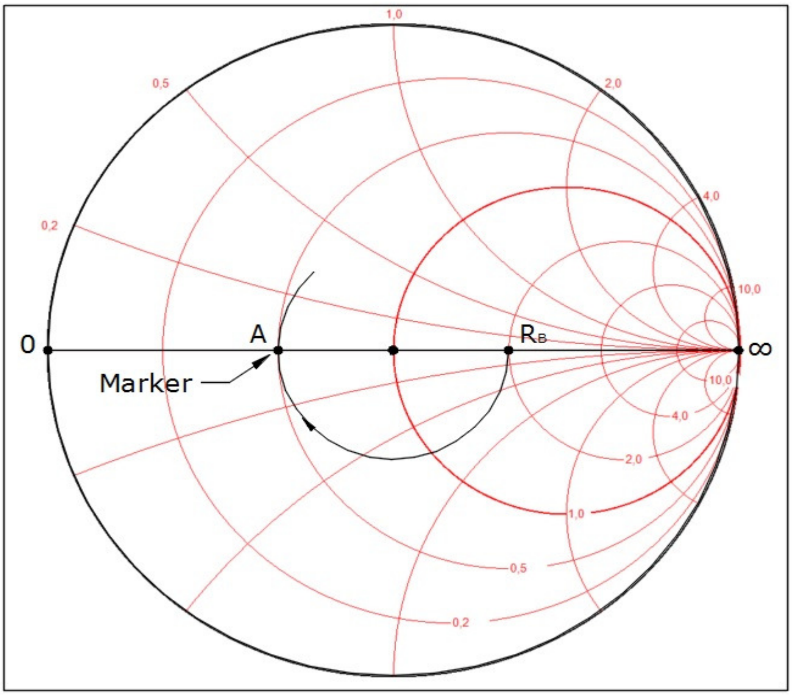

Figure 15-4: Diagram after the scan with VNA.

I would like to point out that in Figure 15-4, the circle does not close because we limited the maximum scanning frequency to f_{MAX}, and the trace will not start exactly at point R_B but slightly after it because the starting frequency of the scan will not be zero.
Now, let's search for point A, indicated by the arrow in Figure 15-4, as it represents the resistance R_A. In searching for point R_A, we need to minimize the imaginary part X_A, although we won't be able to completely eliminate it unless we focus a large number of scan points within the set span. One trick is to reduce the span around point A, but this will require another calibration and scan.
Since we do not need absolute precision, we will settle for finding the value that minimizes the imaginary part as much as possible. This is because the resistor will not be ideal, the connections will not be perfect between the line, the connector, and the resistor, the measurement plane will not perfectly coincide with the calibration plane, and the point R_A will be slightly shifted towards the center due to the attenuation of the non-ideal line. All of these factors contribute to less accurate measurements, so there is no need to strive for perfection with the marker. Approximating values within fractions of pF or pH will suffice.

Once we have centered point A with the marker, we read the value of the real part R_A and its frequency, which we will denote as f_A and express in MHz. Using equation 15.2 and the values of R_A and R_B, we can obtain the characteristic impedance Z_0 of the line.
Furthermore, from the frequency f_A of point A, which I remind you is where the quarter-wavelength transformation occurs, we can obtain the electrical length le of our line segment using a suitably rearranged version of equation 15.3:

$$l_e = \frac{75}{f_A}\text{ m} \qquad 15.9$$

In equation 15.9, the frequency is expressed in MHz, and the length is in meters.
Now, if we explicitly express VF from equation 15.4 and write it in terms of lengths, we have:

$$VF = \frac{l_f}{l_e} \qquad 15.10$$

Thus, with a single measurement, we can obtain two important characteristic parameters of our unknown line.
The third important parameter is derived from equation 15.7 by measuring the magnitude of the reflection coefficient in dB with the line left open and short-circuited at the operating frequency, taken as positive (this is nothing but the return loss). The accuracy of the result is higher with more decimal places and if the line is not significantly shorter compared to the wavelength.

4. Practical example

Now, let's move on to the practical example. With the help of a power drill and the tools shown in Figure 15-5.

Figure 15-5: Tools for wire twisting.

I created a twin-wire line by twisting two 0.5mm enameled copper wires, making approximately 3 twists per centimeter. The line turned out to be 0.372m long, and I connected one end to an SMA-f connector and the other end to a regular 100 Ohm resistor; as shown in Figure 15-6.

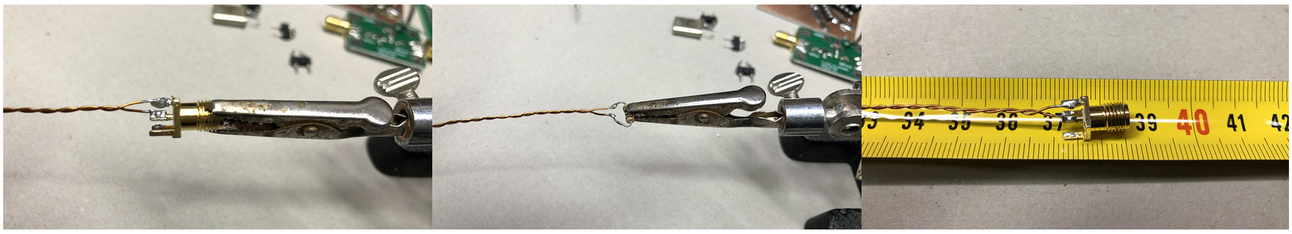

Figure 15-6: Line header and length measurement.

After that, I calculated the maximum frequency to set on the VNA using equation 15.8:

$$f_{MAX} = \frac{75}{l_f} = \frac{75}{0{,}372} = 201{,}6MHz \qquad 15.11$$

I set the stop frequency on the VNA to 250MHz, rounding up, and performed the calibration using the Rosemberger female SOL kit, as shown in Figure 15-7. I also added the delay of the SMA-f connector, which is 84 ps.

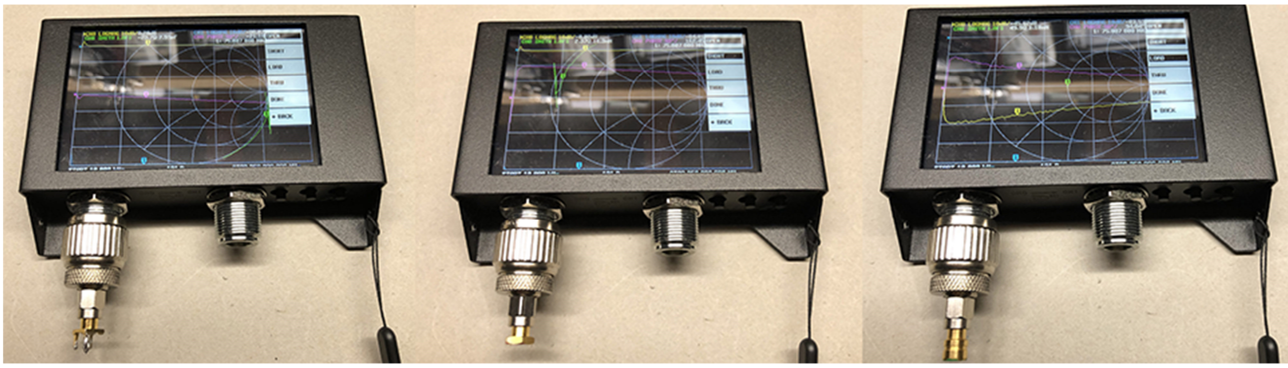

Figure 15-7: Calibration with Rosemberger SOL kit.

After that, I calculated the maximum frequency to set on the VNA using equation 15.8:

Figure 15-8: Scan result with NanoVNA SAA-2N.

Next, I inserted the line and started the scan. The result is visible in Figure 15-8. The scan range is a bit too wide, but in Figure 15-8, we can clearly see the position of the marker (highlighted area A in yellow) and the frequency and impedance values in the circled area in red. Notice the inductance of only 23pH.

The values of interest are R_A=16.2 Ohms and the frequency f_A=125.005MHz. We measured the resistance at the end of the line precisely using a four-wire measurement and obtained R_B=99.26 Ohms, as shown in Figure 15-9.

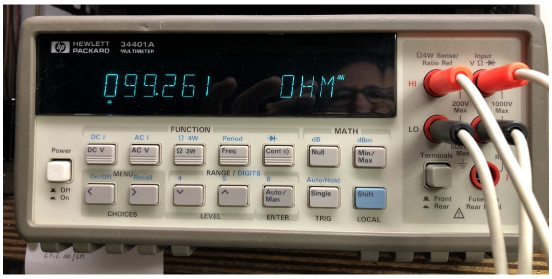

Figure 15-9: Measurement of resistance RB using 4-wire measurement.

Now, let's perform the calculations. Using equation 15.2, we calculate the characteristic impedance:

$$Z_0 = \sqrt{R_A \cdot R_B} = \sqrt{16{,}2 \cdot 99{,}26} = 40{,}01\Omega \qquad 15.12$$

Using equation 15.9, we calculate the electrical length:

$$l_e = \frac{75}{f_A} = \frac{75}{125{,}005} = 0{,}599\text{m} \qquad 15.13$$

Finally, using equation 15.10, we calculate the velocity factor:

$$VF = \frac{l_f}{l_e} = \frac{0{,}372}{0{,}599} = 0{,}62 \qquad 15.14$$

If we want to refine the measurements, we can narrow the span of the NanoVNA to +/-10 MHz around the quarter-wavelength frequency to concentrate the measurement points within the span and increase the measurement resolution. In my case, I will use my DG8SAQ VNWA and concentrate 4000 measurement points in the range from 100 to 150 MHz. The result can be seen in Figure 15-10, and it is very similar to the one obtained with the NanoVNA. The measured values with VNWA (marker 3) are R_A=16.02 Ohms and f_A=125.4 MHz, resulting in:

$$Z_0 = \sqrt{R_A \cdot R_B} = \sqrt{16{,}02 \cdot 99{,}26} = 39{,}87\Omega \qquad 15.15$$

$$l_e = \frac{75}{f_A} = \frac{75}{125{,}2} = 0{,}599\text{m} \qquad 15.16$$

$$VF = \frac{l_f}{l_e} = \frac{0{,}372}{0{,}599} = 0{,}62 \qquad 15.17$$

The values are well within the +/-1% tolerance.

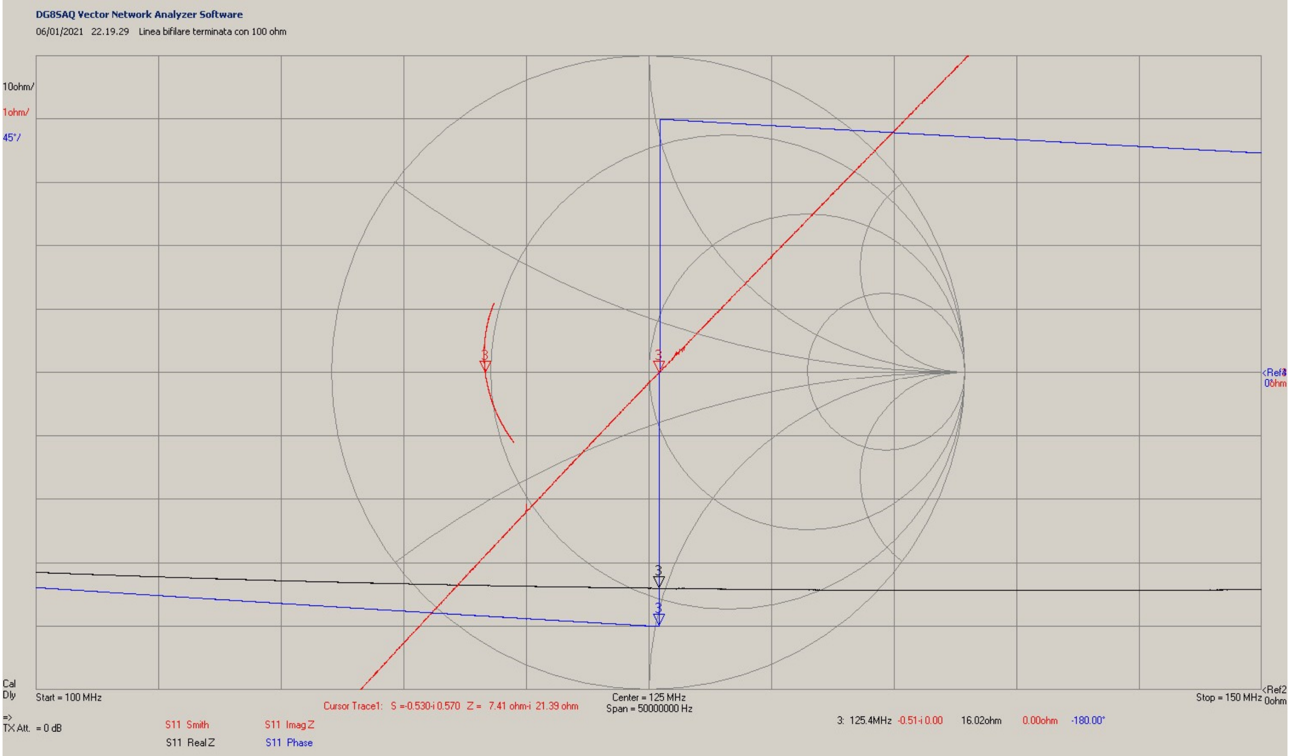

Figure 15-10: Measurement obtained with DG8SAQ's VNWA.

Let's proceed to measure the attenuation using equation 15.7. To do this, we need to measure the magnitude of the reflection coefficient at the operating frequency, which in our case is 100MHz, with an open terminal and a shorted terminal.
First, we remove the 100 Ohm resistor and perform the open scan, as shown in Figure 15-11. In Figure 15-11,

Figure 15-11: Reflection coefficient with open line @100MHz

the value of the reflection coefficient with an open termination is circled in red, and when its sign is changed, it becomes the open return loss RL_O=0.51dB.

Similarly, we perform the scan with a shorted terminal, as shown in Figure 15-12. In Figure 15-12,

Figure 15-12: Reflection coefficient with shorted line @100MHz

the value of the reflection coefficient with a shorted termination is circled in red, and when its sign is changed, it becomes the short return loss RL_S=1.25dB.
Using equation 15.7, we obtain the attenuation of our line segment at 100MHz:

$$L_C = \frac{RL_O + RL_S}{4} = \frac{0{,}51 + 1{,}25}{4} = 0{,}40\text{dB} \qquad 15.18$$

Since the segment length is l_f=0.372m, our line has an attenuation per meter of:

$$A_f = \frac{L_c}{l_f} = \frac{0{,}40}{0{,}372} = 1{,}09\text{dB/m @100MHz} \qquad 15.19$$

This is not negligible.

5. Conclusions

In conclusion, the methods described are not the only ones available, nor are they the most accurate. However, they are practical and allow us to obtain the most important parameters that characterize our lines, even with short line segments. These methods have the limitation of providing valid parameters around the measurement frequency, and the values obtained may vary at different frequencies (especially for self-constructed lines). Furthermore, the accuracy of the measurements depends on the quality of the connections, which, in this example, are quite rough, and the limitations imposed by the instrument in terms of precision. It should be noted that the reflectometer, found in low-cost VNAs, accurately measures impedances four times lower and four times higher than the characteristic value of the reflectometer; i.e., between 12.5 and 200 Ohms.
Overall, I believe that the proposed methods strike a good balance between accuracy and practicality.

6. Bibliography

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2: F. Witt, AI1H: Measuring cable loss, Qex, May/June 2005 p.44
3: L. Attolini: http://www.iw2fnd.it/it/it/content/vector-network-analyzer

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