MAGNETIC INDUCTION IN COAXIAL CABLES

Figure 1 depicts the cross-section of a coaxial cable with a magnetic permeability of μ. Both the central conductor and the shield are made of ideal conductor material with $\mu = \mu_0$. The radius of the central conductor is equal to 'a', the inner radius of the shield is 'b', and the outer radius of the cable is 'c'. The central conductor is assumed to carry the incoming current I_i, while the shield carries the outgoing current I_s.

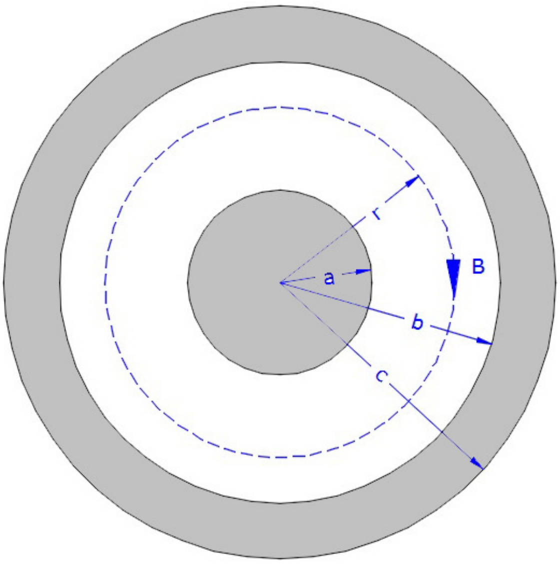

Figure 1: Cross-Section of Coaxial Cable

Here are the main equations we need:

$\oint H \cdot dl = \oint J \cdot dA$ Ampère's Law. 1.1

$B = \mu H$ Relationship between magnetic induction B and magnetic field H. 1.2

$I = J \cdot A$ [A] Relation between current and current density. 1.3

Let's start by substituting equation 1.2 into Ampère's Law 1.1 and obtain:

$\oint B \cdot dl = \mu \oint J \cdot dA$ 1.4

To find the magnitude of the magnetic induction vector B within the cross-section of the coaxial cable, we distinguish cases where the radial coordinate r is:

$r \leq a$ We are inside the inner conductor;

$a \leq r \leq b$ We are in the dielectric;

$b \leq r \leq c$ We are in the outer conductor;

$r > c$ We are outside the coaxial cable.

The direction of the magnetic induction vector B is determined by the right-hand rule and is clockwise. Along the dashed circumference in Figure 1, its magnitude is constant and always parallel to the infinitesimal line dl (Biot-Savart's Law). Therefore, the dot product in equation 1.4 simplifies to a simple product. Similarly, the dot product between the current density and the infinitesimal area *dA* simplifies to a simple product because the current flows perpendicular to the surface A of the conductors.

Therefore, within the inner conductor, the circulation of vector B becomes::

$$B_i \, 2\pi \, r = \mu_0 \, J_i \, \pi \, r^2 \qquad 1.5$$

The current density J_i in the central conductor, from equation 1.3, is:

$$J_i = \frac{I_i}{\pi a^2} \text{ [A/m}^2\text{]} \qquad 1.6$$

We substitute equation 1.6 into equation 1.5 and obtain the value of the magnetic induction vector B magnitude within the central conductor:

$$B_i = \frac{\mu_0 I_i}{2\pi a^2} r \text{ [T] per } r \leq a \qquad \mathbf{1.7}$$

In the dielectric, equation 1.4 becomes:

$$B_d \, 2\pi \, r = \mu \, I_i \qquad 1.8$$

Rearranging, we get:

$$B_d = \frac{\mu \, I_i}{2\pi r} \text{ [T] per } a \leq r \leq b \qquad \mathbf{1.9}$$

Inside the outer conductor, we have:

$$B_s \, 2\pi \, r = \mu_0 \, (I_i \, - \, I_a) \qquad 1.10$$

The current I_a in a outer conductor ring is:

$$I_a = \left(\frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}\right) I_s = \left(\frac{r^2-b^2}{c^2-b^2}\right) I_s \text{ [A]} \qquad 1.11$$

We bring equation 1.11 into equation 1.10 and, simplifying, we obtain the value of the magnetic induction B magnitude in the outer conductor:

$$B_s = \frac{\mu_0}{2\pi r}\left[I_i - \left(\frac{r^2-b^2}{c^2-b^2}\right) I_s\right] \text{ [T] per } b \leq r \leq c \qquad \mathbf{1.12}$$

Outside the coaxial cable, the magnetic induction B is:

$$B_e = \frac{\mu_0}{2\pi r} (I_i \, - \, I_s) \text{ [T] per } r > c \, . \qquad \mathbf{1.13}$$

Let's assume now that the currents I_i and I_s are composed of differential mode currents (CMD) and common mode currents (CMC); Figure 2.

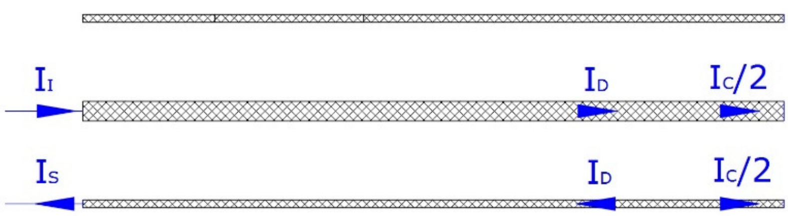

Figura 2: Currents along the coaxial line.

The currents flowing through the central conductor I_i and the currents flowing through the outer conductor I_s can be expressed as follows:

$$\begin{cases} I_i = I_D + \frac{I_C}{2} \\ -I_s = -I_D + \frac{I_C}{2} \end{cases} \qquad 1.14$$

Where the current I_D represents the differential mode current, and the current I_C represents the common mode current. The common mode current I_C, in Figure 2, is evenly distributed in both conductors.

Therefore, outside the coaxial cable, the magnetic induction B from equation 1.13 becomes:

$$B_e = \frac{\mu_0}{2\pi r} (I_D + \frac{I_C}{2} - I_D + \frac{I_C}{2}) = \frac{\mu_0}{2\pi r} I_C \ \text{[T]} \ per\ r > c\ ed\ I_C \neq 0 \qquad 1.15$$

So, in the presence of the common mode current I_C, the magnetic induction B_e outside the conductor is not zero.

However, if the common mode current I_C is zero, the differential mode currents I_D would have equal magnitudes and opposite directions, resulting in zero magnetic induction B_e outside the cable.

$$B_e = 0 \ \text{[T]} \ per\ r > c\ ed\ I_C = 0. \qquad 1.16$$

The distribution of the magnetic induction B magnitude along the coaxial section, in the case where the common mode current $I_C = 0$, is represented in Figure 3.

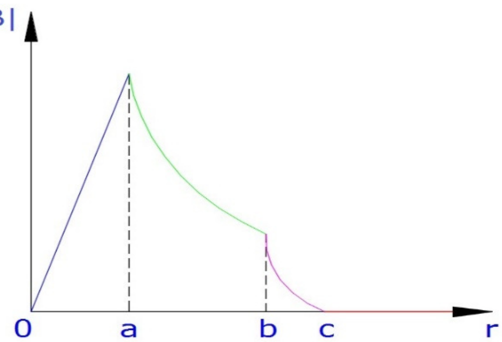

Figure 2: Distribution of magnetic induction B along the cross-section of the coaxial cable with I_C=0.