LET'S DISCOVER TOGETHER THE "CHOKE" OR "CURRENT BALUN"

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Generalities

This is a short article to talk about the function of the device commonly called “choke” or “balun”, but more accurately called “common mode choke”. The article begins with a theoretical introduction followed by a practical example, provided with proofs of effectiveness.

Throughout the article I will take for granted that the currents and the tensions are sinusoidal and you can use phasors to represent them. Also, I'm aware that I have used models with discrete components even though sometimes the physical lengths of the devices are comparable to the wave lengths used.

Common and differential mode currents

In order to tackle the topic pointed out in the title of this paragraph, let's consider a segment of a bifilar line placed in the free space, far from any influence from the ground and whose we don't know neither the provenience nor where it ends; therefore, a completely abstract and imaginary situation [1].

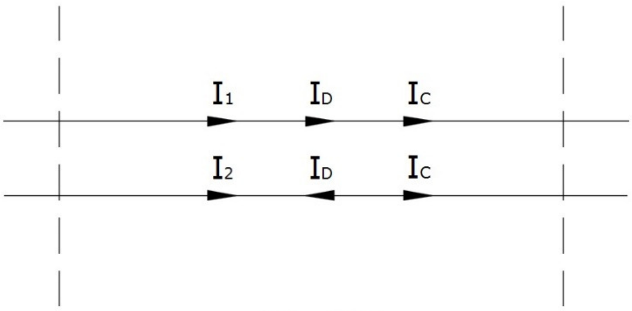

Fig.201

If we were to measure the currents that flow through the two conductors in picture 201, without interfering, we could think that currents I_1 and I_2 are composed by a common mode current I_C (CMC) and by a differential mode current I_D (DMC), linked by the following relations:

$$I_1 = I_C + I_D \quad (2.1a)$$

$$I_2 = I_C - I_D \quad (2.1b)$$

where we imagine that the common mode currents I_C flow through the two conductors in the same direction, while the differential mode currents I_D flow through them in the opposite way. For both currents let's hypothesize that the modulus of I_D are equivalent, as well as the ones of the I_c. This makes the differential currents more intuitive because they can be thought as currents that come from one conductor and go towards the other one, while it's different for the common mode currents they would necessarily need to come back from another way.

The fact remains that, if the wires of the line are close to each other (in relation with the wave length), the differential currents I_D generate EM fields. These fields cancel one

another in the space surrounding the line, while for the common mode currents I_C the fields add up, so they irradiate. I will not dive into this topic any further not to digress too much.
The equations in 2.1 can be rewritten as follows:

$$I_D = \frac{I_1 - I_2}{2} \qquad 2.2a$$

$$I_C = \frac{I_1 + I_2}{2} \qquad 2.2b$$

Let's observe that if the currents I_1 and I_2 were equal in their modulus and opposite in direction, I_1 would be equal to $-I_2$ and from the equations 2.2 we would come up with:

$$I_D = \frac{I_1 + I_1}{2} = I_1 \quad \text{e} \quad I_C = 0 \qquad 2.3$$

The other extreme case is the one where the common mode current I_C is present but the differential mode current I_D is zero.

The ideal common mode choke

The common mode choke, which can be called just choke, is a device that's used to reduce the common mode currents I_C without altering the differential mode currents I_D that flow through the line.

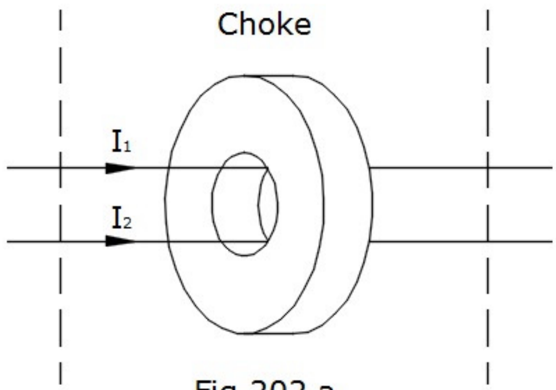

Fig.202 a

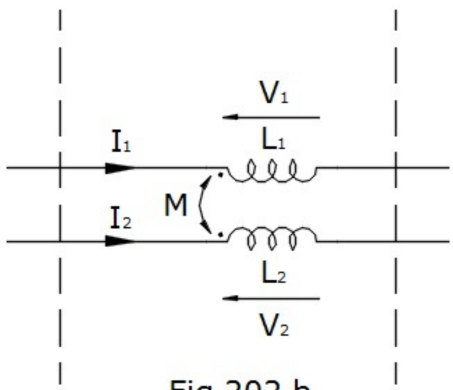

Fig.202 b

The choke is often built just by putting in a ferrite ring in the line, in a way that it coils both the two conductors (fig. 202a), but it can be obtained as well by coiling the line in air (we will get lower inductances). So, as a first approximation, the choke can be displayed with a couple of inductances L1 and L2 mutually paired one to the other M, figure 202b.
As we said previously, if the differential currents I_D that constitute the currents I_1 and I_2 are equal in modulus and opposite in direction (fig. 201), they will generate two equal and opposite fluxes in the ferrite, which cancel one another. Therefore, on the wires of the line in figure 202b the following impedances will manifest:

$$Z_1 = \frac{V_1}{I_1} = \frac{j\omega L_1 I_1 + j\omega M I_2}{I_1} \qquad 2.4a$$

$$Z_2 = \frac{V_2}{I_2} = \frac{j\omega L_2 I_2 + j\omega M I_1}{I_2} \qquad 2.4b$$

Now, if we substitute I1 and I2 with the equations in 2.1 we obtain:

$$Z_1 = \frac{j\omega L_1(I_C + I_D) + j\omega M(I_C - I_D)}{I_C + I_D} = \frac{j\omega I_C(L_1 + M) + j\omega I_D(L_1 - M)}{I_C + I_D} \qquad \textbf{2.5a}$$

$$Z_2 = \frac{j\omega L_2(I_C - I_D) + j\omega M(I_C + I_D)}{I_C - I_D} = \frac{j\omega I_C(L_2 + M) - j\omega I_D(L_2 - M)}{I_C - I_D} \qquad \textbf{2.5b}$$

Then, if we suppose: that L_1=L_2=L, that the pairing between the windings is perfect (k=1) meaning that M=L and the common mode current IC=0, the equations 2.5 will become:

$$Z_1 = \frac{j\omega I_D(L - M)}{I_D} = 0 \qquad 2.6a$$

$$Z_2 = \frac{-j\omega I_D(L - M)}{-I_D} = 0 \qquad 2.6b$$

Basically, the impedance seen by the differential currents is zero. In fact, the fluxes generated by the differential currents in the ferrite are equal and opposite so they cancel each other out; so, the impedance generated by the impedance will be zero. If we suppose again that: L1=L2=L, that the pairing between the winding is perfect (k=1), meaning that M=L, but that this time the differential mode current I_D=0, the equations now 2.5 become:

$$Z_1 = \frac{j\omega I_C(L + M)}{I_C} = j2\omega L \qquad 2.7a$$

$$Z_2 = \frac{j\omega I_C(L + M)}{I_C} = j2\omega L \qquad 2.7b$$

In reality, the impedance seen by the current is not zero at all, in fact it is enhanced by the pairing of the windings. If we had used two identical ferrites, but physically separated, one on each conductor, the mutual pairing would have been null M=0, therefore the factor 2 from 2.7 would have disappeared.

So, as a first approximation, it seems that the choke responds to the expectations: it hinders the common mode currents that flow through the line without influencing the differential mode ones.

The common mode currents in the ferrite choke

Now we will deal with the real ferrite choke. From the practical use of ferrite and from the permeability graphics, it’s evident that the inductive component (μ') dominates. Yet, at the increasing of the frequency the dissipative component (μ") gets more and more prominent; this parameter includes the parasite currents in the ferromagnetic material and the losses by hysteresis. This persists until the reaching of a certain frequency limit, which differs from material to material. If we get past that limit, the ferrite loses all of its ferromagnetic characteristics [2]. The graphic in figure 203 (taken from [3]) shows the permeabilities in function of the frequency of the grade 43 ferrite NiZn (the one that is more suitable in HF amateur bands).

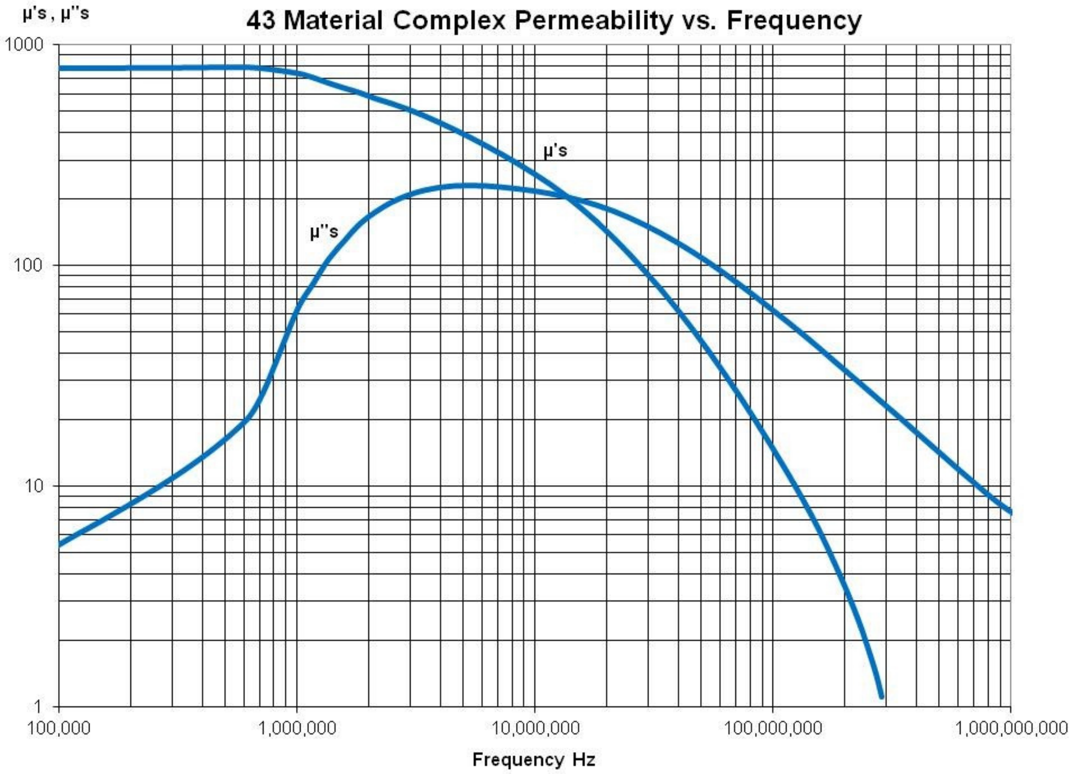

Fig. 203

The dissipative component is usually modeled with a resistance R_S in series to the auto inductance Ls, however, as it is shown in figure 203, both the value of the auto inductance Ls and the one of Rs are strongly dependent on the frequency and on the ferrite mix (besides the temperature of work).

Thus, the choke can be represented by the circuit in figure 204 [4] (that is also the one chosen by the LCR meters):

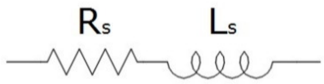

Fig.204

Where $Z_s = R_s + jX_s$ 2.8

The parameters in 2.8 are linked to the ferromagnetic material following the following rule:

$$R_s = \omega L_0 \mu_s^{''} \ , \ X_s = \omega L_0 \mu_s^{'} = \omega L_s \quad \text{where } L_0 = \mu_0 \frac{N^2 A_e}{l_e} \qquad \textbf{2.9}$$

The parameters μ_s' and μ_s'' are respectively the real and imaginary components of the complex permeability, typical of the ferromagnetic material chosen [5]. L_0 is instead the auto inductance that would be present if there wasn't any ferromagnetic material, L_0 depends only on the structure and the square of the number of coils. The auto inductance L of an inductor can also be calculated from the inductance factor A_L multiplied by the square of the number of coils.

$$L = N^2 A_L \qquad \textbf{2.10}$$

with

$$A_L = 4\pi\mu_s' \frac{A_e}{l_e} \text{ [nH/sp}^2\text{]} \qquad \mathbf{2.11}$$

Where AL is in nH/sp^2, the actual area of the ferrite A_e in cm^2 and the actual length of the magnetic circuit l_e in cm; both these two latter parameters are already given by the builder because they are not easy to calculate.
On the contrary, the inductance factor A_L can be easily found because it corresponds to the auto inductance of just one coil turn expressed in nH. Furthermore, when the choke is built with multiple coils winded up, parasite capacities C_P can appear (due to the winded conductor). Their reactance tends to cancel out the effect of the inductance, as the frequency increases. The following model with lumped parameters describes quite accurately a real choke in the range of use of the ferrite.

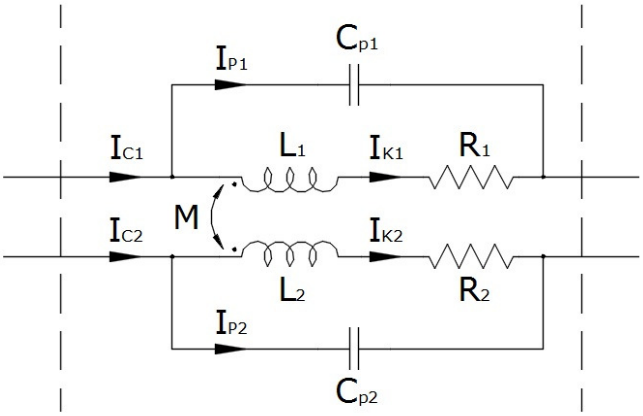

Fig.205

If we hypothesized that the circuit in fig. 205 was symmetric, we could simplify it by substituting the subscript 1 with a 2, so it would end up being: $I_{C1}=I_{C2}$; $I_{K1}=I_{K2}=I_K$; $I_{P1}=I_{P2}=I_P$; $C_{P1}=C_{P2}=C_P$; $L_1=L_2=L$; $R_1=R_2=R$. Assuming this, the impedance seen by the common mode currents will be equal in both the two ramifications and it will be:

$$Z_{1C} = Z_{2C} = \frac{V_1}{I_{C1}} = \frac{j\omega LI_K + j\omega MI_K + RI_K}{I_K + I_P} \qquad 2.12$$

Where I_P is equal to:

$$I_P = \frac{V_1}{X_{CP}} = j\omega C_P (j\omega LI_K + j\omega MI_K + RI_K) \qquad 2.13$$

Which substituted in 2.12 and simplified, putting M=L, generates:

$$Z_{1C} = Z_{2C} = \frac{j\omega LI_K + j\omega MI_K + RI_K}{I_K + j\omega C_P (j\omega LI_K + j\omega MI_K + RI_K)} = \frac{j\omega 2L + R}{1 - \omega^2 2LC_P + j\omega RC_P} \qquad \mathbf{2.14}$$

The equation 2.14 is rather complex, but if we ignore the first-grade term at the denominator $j\omega RC_P=0$ and we transform it in the Bode form, it will be easier to represent it by concentrating on the key points.

$$Z_{1C} = Z_{2C} = \frac{j\omega 2L + R}{1-\omega^2 2LC_P} = R\frac{\frac{j\omega}{\frac{R}{2L}}+1}{-\left(\frac{\omega}{\frac{1}{\sqrt{2LC_P}}}\right)^2+1} \qquad 2.15$$

IMPEDENZA Z_C VISTA DALLE CORRENTI DI MODO COMUNE

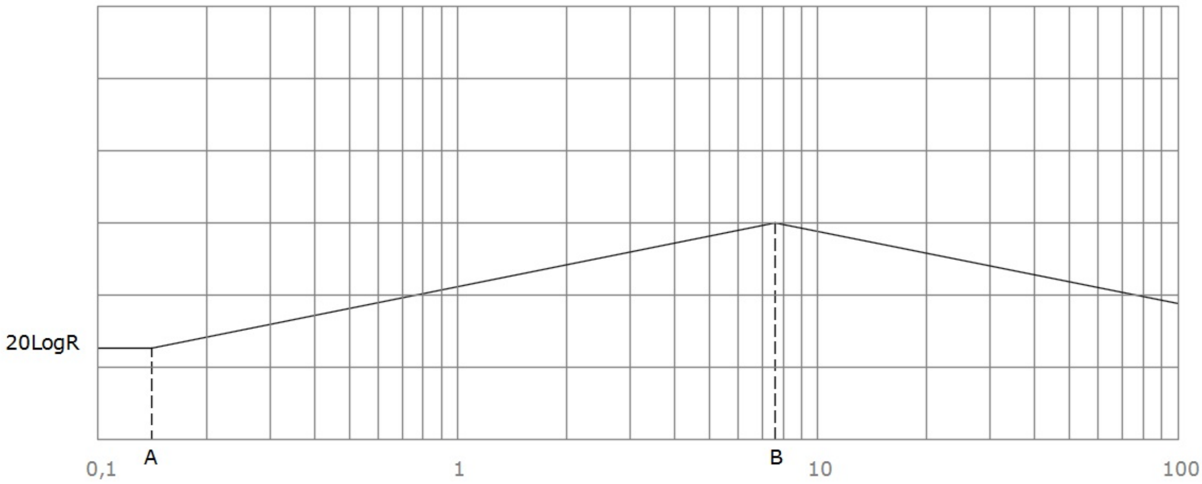

Fig.206

With the increase of the frequency, as shown in figure 206, the impedance appears flat starting from the value 20Log(R) up until reaching point A, where it starts to increase by 20dB/decade; then, it continues to increase until point B, where it starts decreasing by the same value as before, 20dB/decade. It's fairly intuitive that in correspondence to point B there's a parallel resonance (the dampening of which has been disregarded) and that the points A and B correspond respectively to zero and to two poles (I'm only reporting the one in the positive frequency range) which fall under the following f values:

A => $f_A = \frac{R}{4\pi \cdot L}$ 2.16

B => $f_B = \frac{1}{2\pi\sqrt{2LC_P}}$ 2.17

The two points are not easy to calculate because both R and L depend on the frequency (see 2.9) and the parasite capacity C_P is often inscrutable; there are some semi-empirical formulas which try to estimate the auto capacity of a winding, just like the Medhurst formula, but they only take into consideration peculiar solenoids (long, cylindric and in air). In our case, we can use the formula that you can find in [6], which examines the case of windings on a soft ferrite, through the solution of the Maxwell equations with finite elements. In practice, CP can be found by measuring the resonance frequency of an inductor, modeled in the same way as figure 207, with a known capacity in parallel.

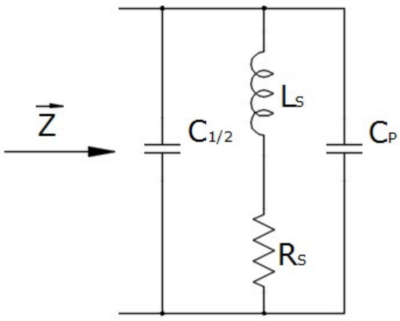

Fig.207

The measure is carried out with a first known capacity C_1 and it's repeated with another known capacity C_2. The two resonance frequencies found with the two capacities f1 and f2 (in MHz) can be combined in the following formula:

$$C_P = C_1 \frac{\frac{C_2}{C_1}\left(\frac{f_2}{f_1}\right)^2 - 1}{1-\left(\frac{f_2}{f_1}\right)^2}$$ with C_1, C_2 and C_P in pF **2.18**

We're going to see later that the curve in figure 206 represents fairly well the range of measurements we are dealing with. As for now, what is our interest is to draw some ideas for the construction. For the matter we observe that point A depends from the relation between the R and L of the ferrite, while point B moves towards higher frequencies the more the denominator gets smaller.
At low frequencies the value of R gets closer to the resistance DC, because the reactance caused by C_P is negligible, therefore the impedance becomes the one described in 2.7. So:

$$Z_{1C} = Z_{2C} = j2\omega L \qquad 2.19$$

In the region between point A and B, the impedance depends from the X and R factors. I would like to remind, however, that R is not exactly a resistor with a well-defined resistance, it is rather the representation of the losses in the ferrite (the ones that heat it up). R represents also the resistances of the conductors (skin effect). So, virtually R is not a resistance, but it behaves like one; for this specific reason it does not have a constant value but it changes at the varying of: the frequency, the permeability μ'' (which also varies at the varying of frequency), the temperature, the structure and the type of ferrite.
A typical example of choke for the common mode currents is shown in figure 208, which is taken from [7]. As you can see (blue trace) reality is different, but it is pretty faithful to the model we considered earlier and even to the simplifications we introduced. In figure 208, point A is visible at around 30MHz (in the previous cases it was at around 10MHz instead). Instead, point B is quite visible at around 900MHz; the presence of a cusp, more or less evident, depends on the dumping factor, the one we ignored by eliminating the first-grade term in 2.14.

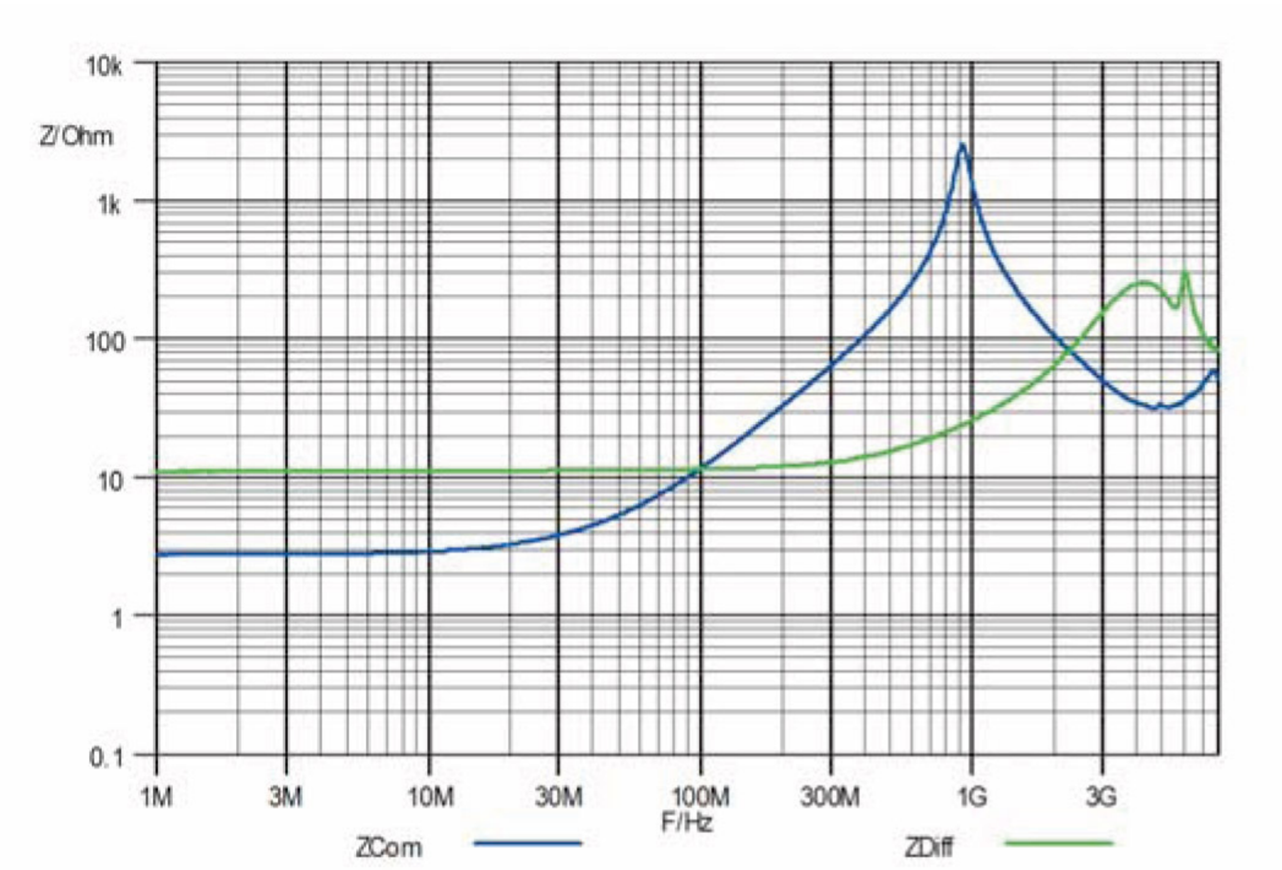

Fig.208

Differential mode currents in ferrite choke

The differential mode currents I_D in the real ferrite choke are not hindered by any significative obstacle because they generate a very small flux inside the ferrite (virtually zero – formula 2.6). This flux is caused by the dispersed fluxes, that cause the coupling to be imperfect, even though they're small, so, in reality k≠1. I'd like to open a parenthesis: the coupling factor k and the auto inductance L_{po} can be easily measured with a RFbridge or with and antenna analyzer. The auto inductance L_{po} is the inductance of the first coil L_1 with the other one opened, while the value L_{pc} can be calculated only from the first coil, but with the second one L_2 is short circuit. Knowing the values of L_{po} and L_{pc} measured at work frequency, with 2.20 we can obtain the coupling factor k:

$$k = \sqrt{1 - \frac{L_{pc}}{L_{po}}} \approx 1 \qquad \textbf{2.20}$$

The value of the mutual inductance M is then drawn from formula 2.21:

$$M = k\sqrt{L_1 \cdot L_2} = k \cdot L \approx L \qquad 2.21$$

With the coupling being imperfect, part of the flux gets dispersed in the surrounding air without interlinking with the conductor coil turns. The said dispersed flux, being in air, will not produce any dissipative effect in the ferrite and it will have a constant permeability close to the one in vacuum ($\mu_0=4\pi 10^{-7}$ H/m). However, the imperfect pairing produces M ≠ L, so 2.6 becomes:

$$Z_{1D} = Z_{2D} = \frac{j\omega I_D (L_s - M)}{I_D} = j\omega(L_s - M) = j\omega L_s (1-k) \approx 0 \qquad \textbf{2.22}$$

The difference between L_s and M is very small (k is often over 0.99) so the effects on the differential currents can be seen only at very high frequencies, way over point B of figure 206. At those frequencies even the effects due to the parasite capacities and the resistance of the conductors (due to skin effect) can manifest (the losses in the dielectric in HF are still negligible). Hence, I suggest again a model similar to the one in figure 205.
Therefore, the impedance seen by the differential current will have, in the range of interest, the resistance in series, generated by the skin effect R_w, which in the case of copper conductors is:

$$R = 83{,}04 \cdot 10^{-6} \frac{l_w}{d_w}\sqrt{f} \quad \text{with } l_w, d_w \text{ in mm and f in MHz} \qquad \textbf{2.23}$$

Thus, the parasite capacity associated to the inductance loss will generate a resonance in parallel at way higher frequencies compared to point B, which we encountered talking about the common mode currents and which is usually not in the range of use of the choke (see the green trace in figure 208).

Common mode currents in a circuit

Now let's take into consideration what happens to the common mode currents (CMC) when we insert a ferrite choke in a transmission line, which links a generator to its load.

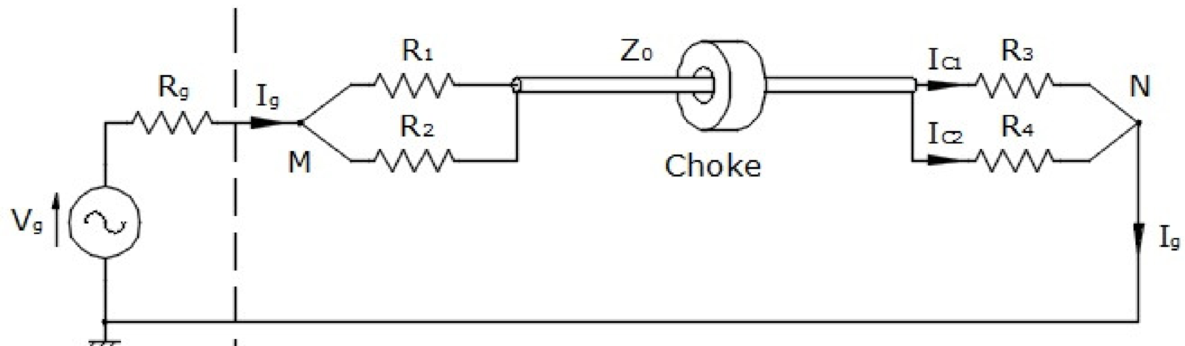

Fig.209

The test circuit proposed is the one in figure 209. Given that the resistances $R_1=R_2=R_3=R_4=50\Omega$ are in series on two different paths and the two are in parallel one with the other, the generator will make the current I_g flow and it will generate a resultant of R=50Ω. Furthermore, given that the two paths are also considered as symmetrical, the currents $IC_1=IC_2=I_g/2$ can be considered common mode currents because they are equivalent both on the inner conductor and the braided outer conductor. Lastly, an equivalent electric circuit will come out like the one in figure 210.

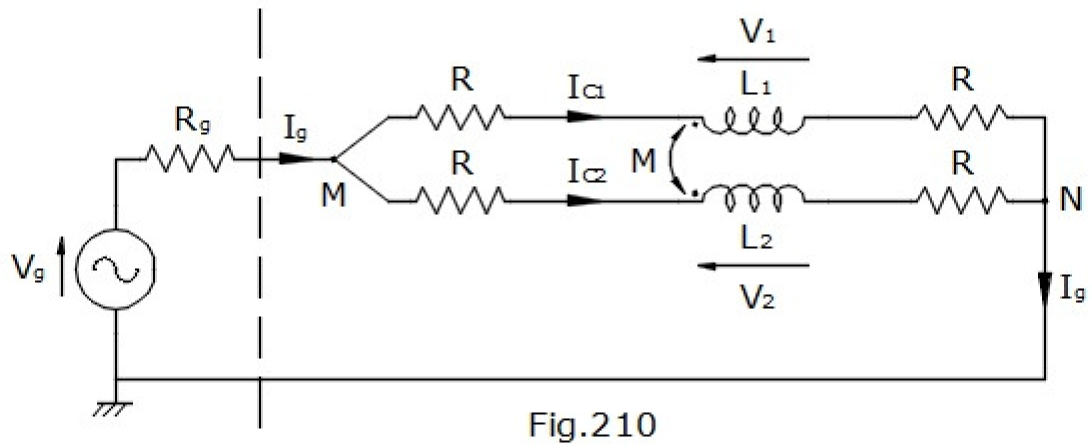

Fig.210

The circuit in figure 210 is very useful to measure the common mode currents I_{C1} and I_{C2} (they can be calculated from the voltage drop of the respective R) and the drops in voltage in the line segment V_1 and V_2. Then, if we use a "clip" type ferrite, we can see the influence that it has on the common mode currents with or without the ferrite plugged in.
Let's hypothesize that the inductances produced by the ferrite are equal $L_1=L_2=L$ (in the case of a coaxial line this is not exactly true, but it is acceptable) and that the coupling between the two is perfect (so that M=L); this is reasonably true for the coaxial lines and less true for the bifilar lines. This way we will obtain:

$$Z_1 = j\omega L_1 + J\omega M = j\omega 2L \qquad 2.24a$$

$$Z_2 = j\omega L_2 + J\omega M = j\omega 2L \qquad 2.24b$$

The tension at the ends of the two inductances is equal, $V_1=V_2=V$, because they are in parallel with each other. Therefore, the voltage drop at the ends of the inductances will be:

$$V = j\omega 2LI_C = j\omega LI_g \qquad 2.25$$

Let's carefully observe 2.25: it basically tells us that the flux Φ_T generated in the ferrite by the common mode currents I_{C1} and I_{C2} is equal to the flux that the current I_g would generate if it flowed by itself through a single conductor.
You might say it is obvious, because $I_g=I_{C1}+I_{C2}$ but the repercussions of this are not so obvious.
The losses in the ferrite (R_S) are caused by the flux Φ_T generated in the ferrite by the current I_g (in the term R_S are included the losses in the conductors, which are considered negligible); in addition, the transmission line can be substituted with a conductor which is run through by the total current I_g. This latter simplification, which can be obtained by soldering the two conductors of the line one with the other, allows us to simplify the study of our choke if we ignore the losses caused by the skin effect in the conductors (which in the case of a coaxial line are slightly different between one another).The parasite capacity C_P, which limits the bandwidth of the choke, is equally present and, if we use the coaxial in short circuit at the ends, it practically remains the same.
In other words, we can use our model with concentrated parameters of figure 211 to study our choke, because it approximates quite well the reality of the HF bands.

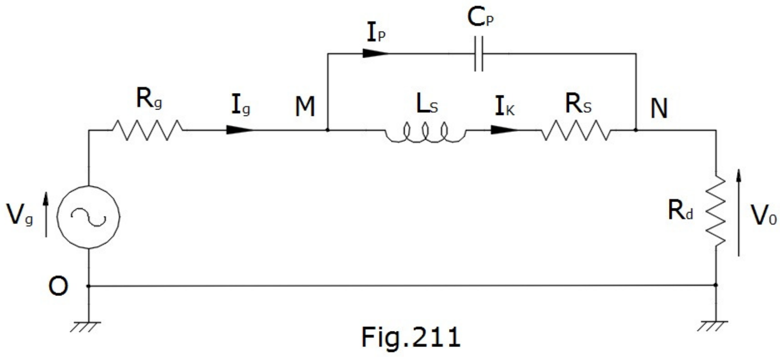

Fig.211

In the circuit in figure 211 the auto inductance L_S=L and the losses in the ferrite R_S are shown (the subscript $_S$ stands for series model); both parameters are strongly dependent on the frequency (see figure 204 and equations 2.9). The losses caused by the skin effect in the conductors of the coil are plugged into the R_S (in our case the losses in the ferrite are way higher than the ones in the conductors). Ultimately, the parasite capacity C_P is considered independent from the frequency, as we have seen in figure 205 (this is quite true, as long as the frequency stays under the auto resonance frequency).
It is a good thing to calculate the scatter parameter S_{21} from the model in figure 211 [8] because it is what an electrical analyzer measures and it is tightly related to the insertion loss IL. So:

$$IL = -20 Log|S_{21}| \text{ dB} \qquad \textbf{2.26}$$

As we can see from 2.26 the insertion loss IL in dB represents $|S_{21}|$ in dB in a specular way as regards to the frequency axis (S_{21} is, actually, the reciprocate of IL and that is rendered with the minus sign before the logarithm).
To calculate the IL it is useful to repeat the definition of insertion loss and express it in current.

$$IL = 10 Log\left|\frac{P'}{P}\right| = 10 Log\left|\frac{I_g^{2'} R}{I_g^2 R}\right| = 20 Log\left|\frac{I_g'}{I_g}\right| \text{ dB} \qquad 2.27$$

Where P' in the power dispersed on a hypothetical load R without a choke, while P is the one we would have on the load with the choke plugged in.
The circuit model in figure 211 it is the same type as the ones shown in figure 212; see [1].

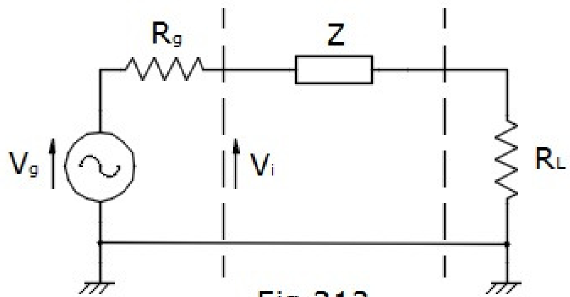

Fig.212

Therefore, the insertion loss caused by the impedance Z turns out as follows:

$$IL = 20Log\left|1 + \frac{Z}{R_g + R_L}\right| \quad 2.28$$

The impedance Z is already known from 2.14. If we put L_s=2L and R_s=R (I would like to remind that R_s represents the losses in the ferrite, L_s the inductance of the coil with the ferrite and C_P is the sum of the parasite capacities; see 2.9); so we obtain:

$$IL = 20Log\left|1 + \frac{1}{R_g + R_L}\left(\frac{R_S + j\omega L_S}{(1-\omega^2 L_S C_P) + j\omega C_P R_S}\right)\right| \quad 2.29$$

Basically, we find again the curve traced in figure 206, divided by the R_g+R_L and summed to the unit. The main points A and B are still the same as the ones in 2.16 and 2.17:

A => $f_A = \frac{R_S}{2\pi L_S}$ **2.30a**

B => $f_B = \frac{1}{2\pi\sqrt{L_S C_P}}$ **2.30b**

I'm writing again the equation of the module for completeness:

$$IL = 20Log\left[1 + \frac{1}{R_g + R_L}\sqrt{\frac{{R_S}^2 + \omega^2 {L_S}^2}{(1-\omega^2 L_S C_P)^2 + (\omega C_P R_S)^2}}\right] \quad 2.31$$

From 2.31 we can easily find that in DC we have R_S=L_S=C_P=0, so:

$$IL_0 = 20Log(1) = 0 \quad 2.32$$

At high frequencies, where the reactance C_P prevails, we have a low R_S and L_S (because µ' and µ" drop) and they are bypassed by the parasite capacity. So, we obtain:

$$IL_H \approx 20Log(1) = 0 \quad 2.33$$

At lower frequencies L_S prevails, while C_P and R_S are not yet visible, we have:

$$IL \approx 20Log\left(1 + \frac{\omega L_S}{R_g + R_L}\right) \quad 2.34$$

Far from the extreme values, the curve is similar to the one already seen in figure 206. Ultimately, it is possible to obtain the equation of the impedance Z seen between the two broken lines in figure 212:

$$Z = \frac{R_S + j\omega L_S}{(1-\omega^2 L_S C_P) + j\omega C_P R_S} \quad \textbf{2.35}$$

Differential mode currents in a circuit

Now let's examine what happens to the differential mode currents (DMC) when we insert a ferrite choke in a transmission line which links a generator to its load.

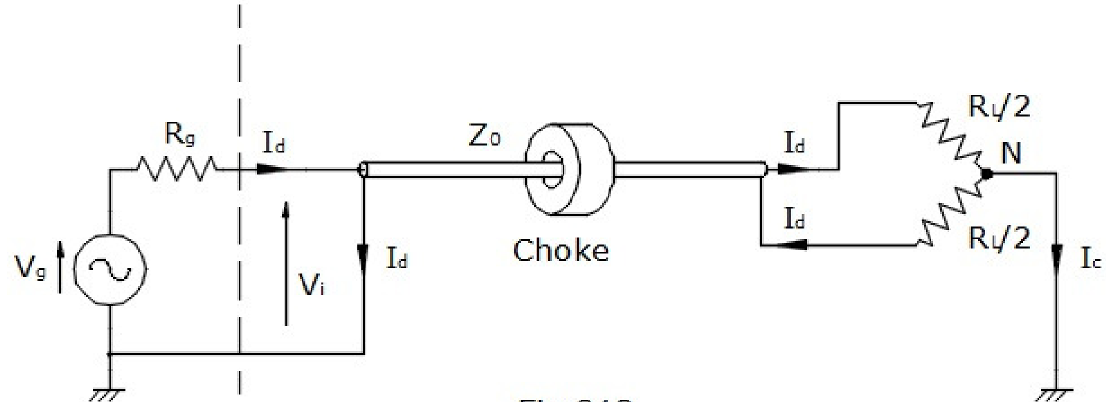

Fig.213

From figure 213 we can observe that the RF generator, unbalanced towards the ground because of the current I_d, which enters into one of the two conductors of the line (in this case in the coaxial inner conductor), it flows through the first half of the load up until the node N and, if I_c=0, it comes back from the braiding of the coaxial, continuing towards the mass of the generator V_g. In this case the current I_d is entirely a differential mode current; its module does not change throughout its path. All of this will happen if the current I_c is zero and/or if the segment that links the node N to the ground is absent; this way we can make a floating load as shown in figure 214.

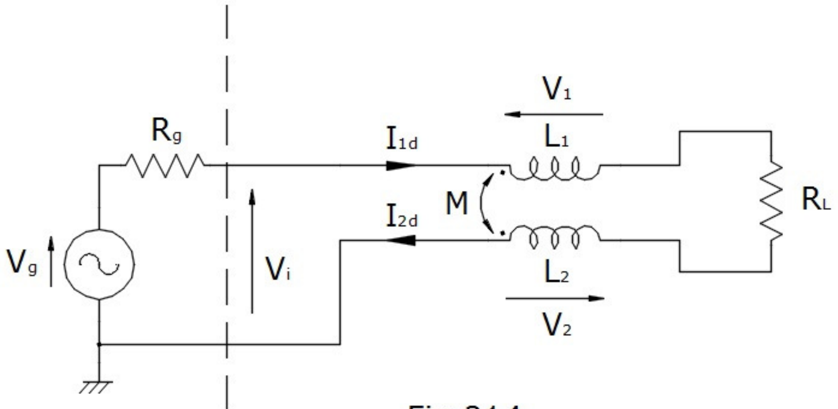

Fig.214

As we have already seen in the beginning paragraph (equation 2.22), in the case that the current was equal to zero and the pairing between the two coils was almost perfect (k≈1), the inductances L_1 and L_2 seen from the DMC would be very small, because they are generated from the dispersed fluxes and, in series to the differential mode current $I_{D1}=I_{D2}=I_D$. As a result, if we suppose: $L_S=L_1=L_2$, $R_L=R_1+R_2$, we will have:

$$Z_D = j2\omega(L_S - M) = j2\omega L_S(1-k) \approx 0 \qquad \textbf{2.36}$$

Therefore, the impedance seen by the differential mode current I_D, when $I_C=0$, is very small and comparable to the loss in the conductors (expressed in 2.23); but these are minor too, so, in the configuration in figure 214, the generator basically only sees the load R_L. On the contrary, at very high frequencies the dispersion inductances of 2.36 start to increase and get heavy, as shown in figure 208.

In the case in which the common mode current I_C, in figure 213, was not zero, the situation would be like the one in figure 215.

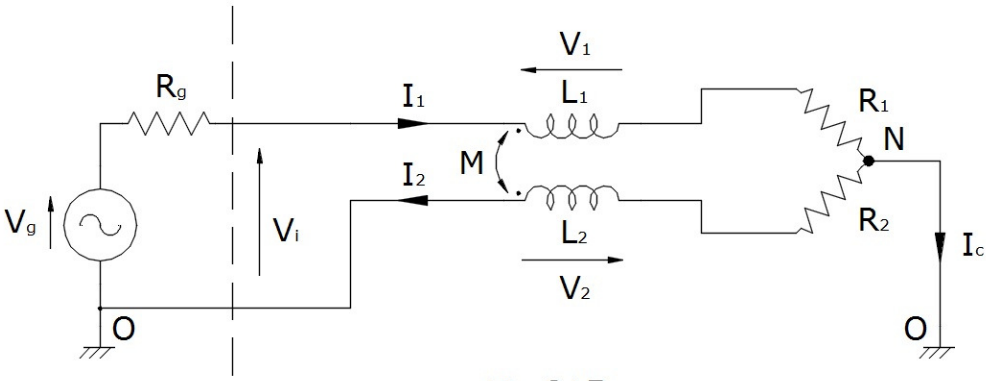

Fig.215

In this latter case the current I_C will be:

$$I_C = I_1 - I_2 \neq 0 \qquad 2.37$$

From which we can draw what follows:

$$I_1 \neq I_2 \neq I_d \qquad 2.38$$

Given it is symmetrical, the circuit in figure 215 can be modelled in the following way:

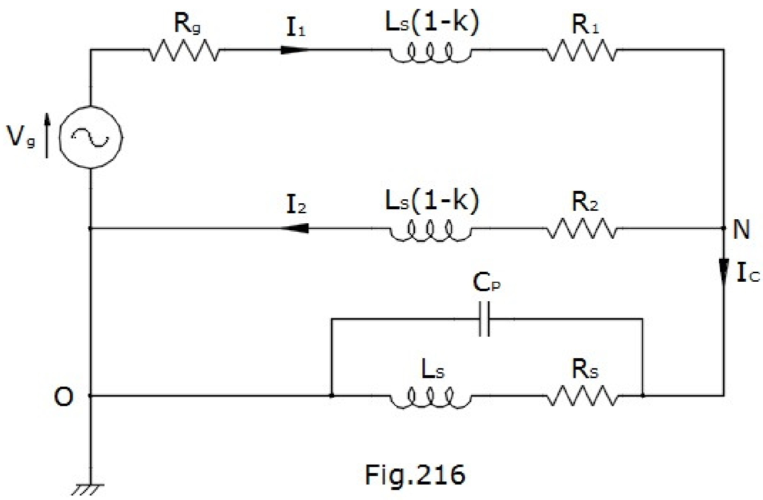

Fig.216

Referring to figure 215, in the pathway between the node N and the ground, only the common mode current I_c will flow (presumed to be of conduction) and it will be hindered solely by the impedance generated by L_s, R_s e C_p (figure 216). Hypothesizing that the currents I_1 and I_2 in figure 216 are:

$$I_1 = I_d + \frac{I_c}{2} \qquad 2.39a$$

$$I_2 = I_d - \frac{I_c}{2} \qquad 2.39b$$

Then, applying the superposition of effects, it is evident that:

- w
  hen I_c=0 the currents $I_1 = I_2 = I_d$ and the current I_d flows only through the impedance Z_d:

  $$Z_d = (R_1 + R_2) + j2\omega L_S(1-k) \approx R_1 + R_2 \qquad 2.40$$

  The “approximately” is mandatory because the pairing factor is k ≈ 1, so that it cancels out the imaginary term in 2.40.

- when I_d=0, only the current I_c will flow, the latter being limited by the impedance Z_c (Z_c is again the one from 2.35).

  $$Z_c = \frac{R_S + j\omega L_S}{(1-\omega^2 L_S C_P) + j\omega R_S C_P} \qquad 2.41$$

Considering 2.40, the circuit in figure 216 transforms into the circuit that follows:

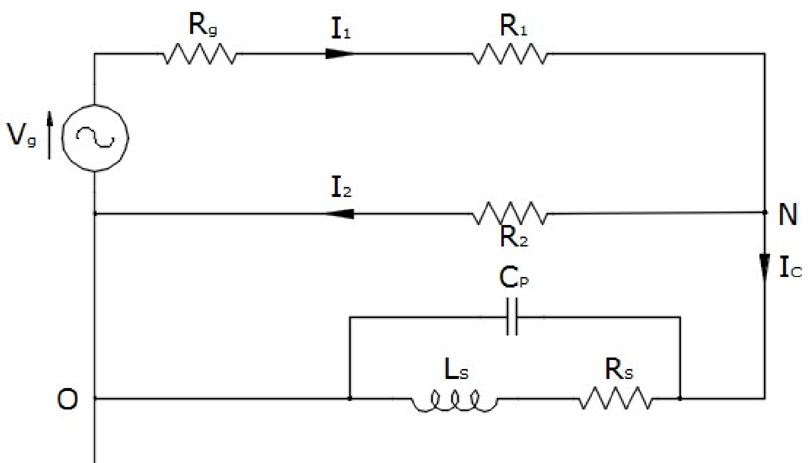

Fig.217

From the circuit in figure 217 it is even more noticeable that the system L_s, R_s and C_p (whose impedance Z_c is drawn from 2.35) is in parallel to the resistance R_2. This is not particularly ideal because when the impedance is not high enough, the differential mode current does not come back entirely through R_2; this also happens when the load $R_1 + R_2$ is adapted to the line. Therefore, if the generator V_g, with its internal resistance R_g, saw a perfectly paired load (hence $R_g = Z_0 = R_1 + R_2$ with $R_1 = R_2$), it would immediately mismatch when the central point N is grounded; all of this is due to the fact that the resistance R_2 is connected in parallel with Z_c. This mismatch will make itself more and more felt the more the module of Z_c decreases, until Z_c will cause a short circuit in R_2 letting all the current I_1 flow through R_1 and come back to the grounding as a common mode current I_c. Clearly, $I_1 = I_c$ is exactly the situation we want to avoid, so it is necessary that the value of the module of Z_c is as great as possible, this way it will have the littlest influence in parallel with R_2. This condition, in practice, is already satisfied with Z_c values that are at least ten times greater than R_2:

$$|Z_c| \geq 10R_2 \quad \textbf{2.42}$$

The magnitude plot for the impedance of the system is marked out in figure 206. It appears evident that there are two values of frequency which mark the boundary inside of which 2.42 is satisfied. I will conclude here, to prevent an already loaded paragraph from becoming too dense of information. If someone was interested in knowing more you could always reach out to me for clarifications or additional information.

Considerations on power

When a ferrite choke is working it creates a flux Φ which hinders the flowing of the common mode current; the consequence is a voltage drop at the ends of the conductors that go through the choke. The manageable power of a given choke basically depends on three elements: the losses in the conductors, the losses in the ferrite and the absolute maximum flux. The losses in the conductors of the line at high frequencies make them heat up due to Joule effect. Joule's law states that the heat produced is $Q=R \cdot I_C^2 t$ (where R is the resistance, I_C the CMC current and t time), where the R of the conductor at high frequencies increases by the increasing of the square root of the frequency of work (see 2.23). Instead, the losses in the ferrite are mainly caused by the parasite currents and by the losses by hysteresis. Said losses produce heat and they are represented in our models by the resistor R_S. The parasite currents are directly proportional to the frequency of use (and also to the permeability and are influenced by other factors depending on the material chosen), while the losses by hysteresis, for the same frequency, are directly proportional to the amplitude of the common mode currents that supply the flux.
On the other hand, the maximum flux in a ferrite has to be limited because if it exceeds the saturation point, the relative permeability of the ferrite drops and reaches values close to the ones of air ≈1 and this causes non linearity which generates distortions and harmonics. Unfortunately, it is very hard to estimate the common mode current that gets halted and because of that neither the CMCs, nor the voltage drop that will manifest at the ends of the choke will be known beforehand; the differential mode current (DMC) that the line has to carry will be known, instead, but it only serves to size the conductors.
Nonetheless we can still make some considerations: the highest dispersion of power happens at higher frequencies, therefore we will have to carry out the power stress tests at the highest frequencies of use; the more heat is produced, the more there will be the need to have it dispersed in the environment, so in order to prevent the reaching of the Curie point, it is necessary to maintain the losses in the ferrite as low as possible. In addition, we need to make sure that the potential heat that is produced can be easily dispersed in the environment.

Building a choke

After all of this theory we need to put something into practice, otherwise we are going to lose the practical sense of things.
I will now show you, step by step, how to build a choke in a coaxial cable insertable in a transmission line (always coaxial) which supplies a balanced load. We are also going to make sure that it reduces the common mode currents of at least -20db in the band 1,83 ÷ 28 MHz.
As far as we know, the most critical working point for the choke is when it works at low frequencies, while the passband is limited to the high frequencies by the parasite capacities C_P; we will try to minimize the latter by reducing as much as we can the number of spires and by distributing them all over the ferrite. Since we want a reduction of -20dB, we deduce from 2.26 that the insertion loss caused by the choke will have to be IL=20dB.
So, rewriting 2.28, we obtain:

$$|Z| = (R_S + R_L)\cdot\left(10^{\frac{IL}{20}} - 1\right) \qquad \textbf{2.43}$$

Usually the spectrum analyzer has $R_S=R_L=50\Omega$; so, if we set IL=20dB in 2.43 we obtain:

$$|Z| = 100\cdot\left(10^{\frac{20}{20}} - 1\right) = 900\ \Omega \qquad 2.44$$

Let's hypothesize that we want to use a ferrite FT240-43. From figure 203 we can gather that the dominant permeability at the lowest frequency f_L is $\mu_s'\approx620$. So, we can think of the desired |Z| as solely consisting of the inductive component, knowing that this is considered a worse simplification of reality. In a real situation we will also have, in fact, the resistive component R_S which comes to the aid of our choke. Therefore, at the frequency of work f_L=1,83MHz we can consider:

$$|Z| = X_S \qquad 2.45$$

where:

$$X_S = \omega L_S = 2\pi f N^2 A_L \quad \Omega \qquad 2.46$$

A_L can be obtained from 2.11 and from the magnetic features of the ferrite FT240-43

Ferrite FT240-43		
A_L	1075 ±20%	nH/sp^2
A_e	1.58	cm^2
l_e	14.5	cm
V_e	22.8	cm^3
B_{max}	2900	G
T_C	>130	℃

Tab.1

$$A_L = 4\pi\mu_s'\frac{A_e}{l_e} = 4\pi\cdot620\frac{1{,}58}{14{,}5} = 849\ \text{[nH/sp}^2\text{]} \qquad 2.47$$

Notice that the tolerance indicated in the building characteristics is +/-20%; for this particular reason I prefer to measure the actual A_L of the ferrite that I intend to use. From the graphic that I got measuring the S_{11} of one spire winded on the ferrite that I chose for my test (with VNWA by DG8SAQ [9]) I obtained the auto inductance of a spire, which corresponds to the A_L (colored in blue in the graphic).

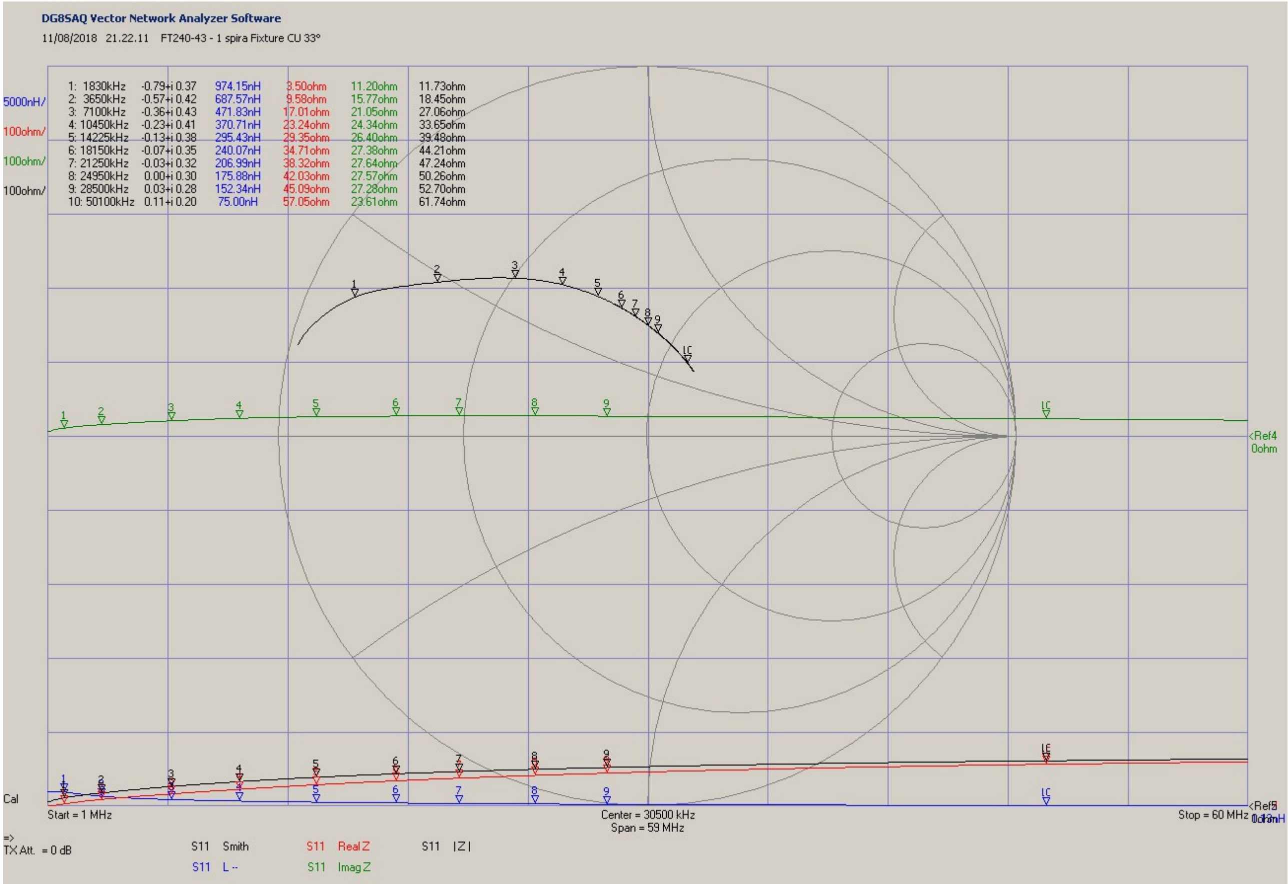

Fig.218

For convenience I included all the values in table 2.

FREQUENZA	A_L (nH/sp^2)	Rs (Ω)	Xs (Ω)	Zs (Ω)
1,830	974	3,5	11,2	11,7
3,650	687	9,6	15,8	18,5
7,100	471	17,0	21,1	27,1
10,450	370	23,2	24,3	33,7
14,225	295	29,4	26,4	39,5
18,150	240	34,7	27,4	44,2
21,250	207	38,3	27,6	47,2
24,950	175	42,0	27,6	50,3
28,500	152	45,1	27,3	52,7
50,100	75	57,0	23,6	61,7

Table 2

As you can see the value of A_L is 974nH/sp^2, which is about 15% higher than the one calculated. From 2.46, establishing as the value of A_L the value we can find in table 2, we obtain the minimum number of spires that we should have:

$$N = \sqrt{\frac{|Z|}{2\pi f A_L}} = \sqrt{\frac{900}{6{,}28 \cdot 1{,}83 \cdot 10^6 \cdot 974 \cdot 10^{-9}}} = \sqrt{\frac{900 \cdot 10^3}{6{,}28 \cdot 1{,}83 \cdot 974}} = 8{,}96 \approx 9 \text{ Spires} \qquad \textbf{2.48}$$

Keeping in mind as a reference the 9 spires obtained from 2.48, let's verify if X_s remains over 900Ω all throughout the useful bandwidth, always being conscious of the approximation by defect when applying 2.46. So, substituting to A_L the values in table 2 we will find:

f (MHz)	1,830	3,650	7,100	10,450	14,225	18,150	21,250	24,950	28,500
A_L (nH/sp^2)	974	687	471	370	295	240	207	175	152
Xs (Ω)	907	1276	1701	1967	2135	2216	2238	2221	2204
IL (dB)	20,06	22,77	25,11	26,31	26,98	27,29	27,38	27,31	27,25

Table 3

From the data in table 3 we can see that X_s always remains over 900Ω and that the IL remains always over the 20dB required all throughout the bandwidth. It seems like everything is going smoothly, even if we only estimated the inductive component to which the resistive component R_s will come to the aid (the one that only takes into account the losses in the ferrite). Yet, all of this will be partially eroded by the parasite capacity C_P, starting from the frequency in point B. This parasite capacity is, unfortunately, very difficult to estimate before having actually built the choke.
But now, let's move onto winding the choke.
To do that, let's use the ferrite FT240-43, the same we used before for our measurements, and a coaxial cable which is able to handle the differential mode current (the one which supplies the balanced load). For the purpose, I used a deformable coaxial cable SM141-50 I bought in lengths of 1.2m (3ft) at Friedrichshafenhamfest. After checking if it was 50 Ω, I looked up its specifics online [10]. The deformable coaxial cable, compared to the flexible (RG142B/U) ends up being: better adaptable, with a slightly smaller diameter and able to exceptionally maintain the shape once winded. The specifics of the cable SM141-50 are listed in table 4.

SM141-50		
Z_0	50	Ω
K	0,695	
Diametro esterno	4,14	mm
Tipo d'isolamento	PTFE	
Raggio min di curvatura	8	mm
P_{MAX} @10MHz	3,45	kW
V_{MAX}	1,9	kVrms

Table 4

It is a very well-made cable and it is able to handle way more than the standard 500W allowed to us Italian OMs.
Now we need to find the average length of a spire.
The simplest and quickest method consists in winding a spire of the chosen coaxial cable, mark the intersection point on both parts of the cable with a marker and then measure the distance between the two marks. In our case the average spire measures l_{wm}=70mm. Then, we need to obtain the length of the line, making sure to add an extra spire for the connections.

$$l_{WP} = l_{wm} \cdot (n_P + 1) = 70 \cdot (9 + 1) = 700 \quad \text{[mm]} \qquad \textbf{2.49}$$

Having calculated the number, what we need to do is rather simple: we need to wind four spires on one side and the remaining four on the other side, in order to have the input on one side and the output on the opposite. The total number of spires that are winded corresponds to the number of times the coaxial dives into the central hole; precisely 4+1+4=9 spires (with the inversion, represented by the number 1, they always come out as an odd).

Fig.219

Once the choke is built as shown in figure 219 and ready for connection, let's move onto the testing. Let's connect the choke to the network analyzer, as in figure 212; V_g and R_g are inside the tracking generator TX, while the resistance R_L comes from the detector RX (the broken lines denote the limit points of the device). Then, let's measure the parameter $|S_{21}|$ in dB and represent it on a logarithmic scale of frequencies, so that we're able to compare the results with the Bode diagram of figure 206. I would like to remind that from 2.25 we can draw $IL=-20Log|S_{21}|$; so $|S_{21}|$ is equal to the IL, but specular compared to the graphic in figure 206. The measure is carried out with inner and outer conductors joint together. The result is shown in figure 220.

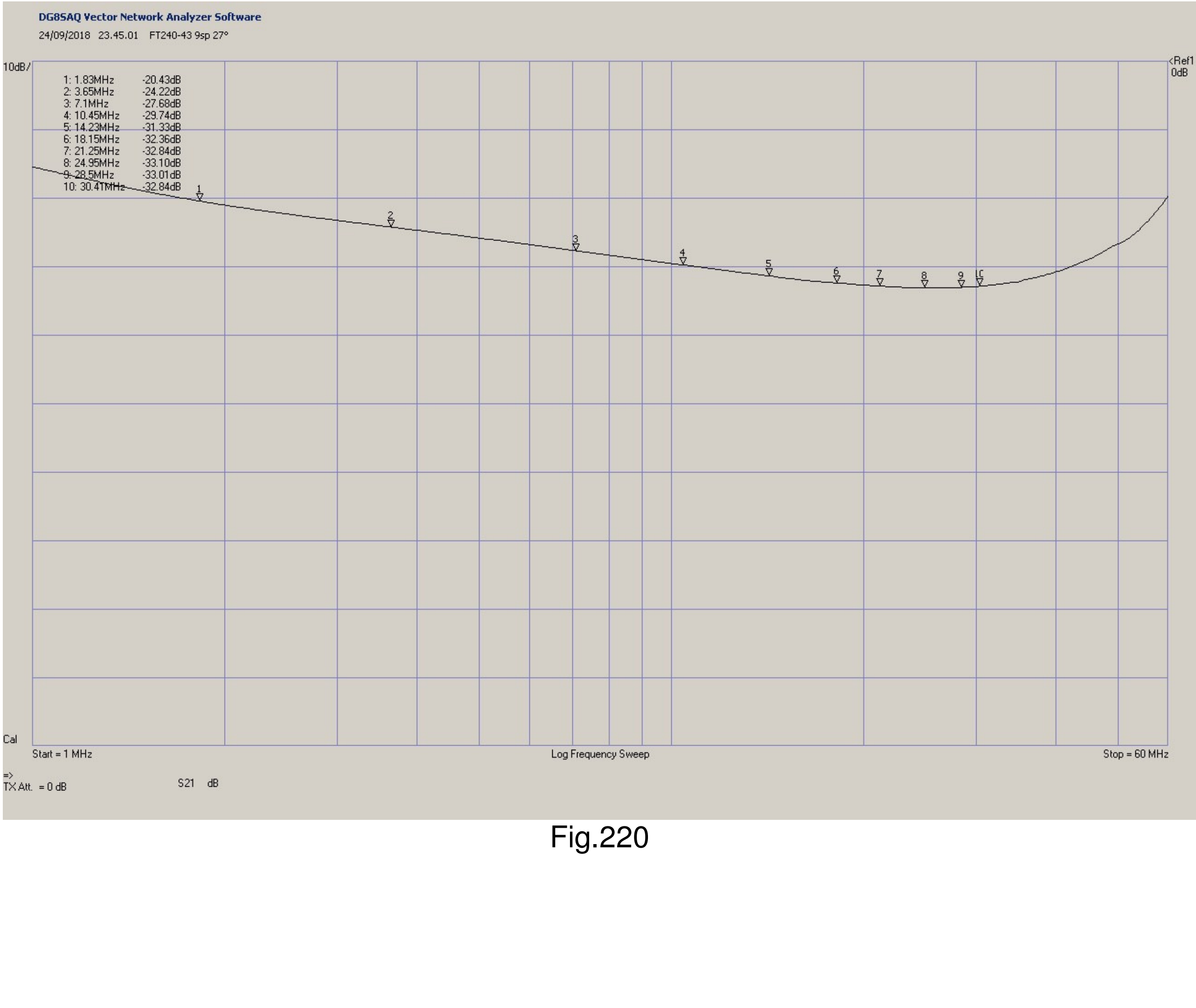

Fig.220

The marker n°10 represents point B, the same there was in figure 206, which corresponds to the resonance caused by the parasite capacity C_P. The frequency of point B is f_B=30,41MHz, while point A is out of scale at about 18KHz (at 18KHz Rs≈0 and µ'=800). For convenience I am reporting the values of IL of table 5.

f (MHz)	**1,830**	**3,650**	**7,100**	**10,450**	**14,225**	**18,150**	**21,250**	**24,950**	**28,500**
IL (dB)	20,43	24,22	27,68	29,74	31,33	32,36	32,84	33,10	33,01

Table 5

It's surprising how the value calculated at 1.83MHz is so close to the one measured. Yet, the initial segment, up until point B, slopes by less than -20dB/dec, while the segment that follows point B is way steeper than the +20dB/dec expected. The graphic resembles the specular image of the one in figure 206, without considering the slopes. This comforts us on the good built of the circuital model, even though it does not take into account the parameters R_S and L_S, which vary with the frequency.

Just out of curiosity, let's calculate the value of C_P we obtain winding the 9 spires in the way you can see in figure 219. Let's calculate the inductance L_S at point B from 2.10 and 2.11 (f_B=30,41MHz) and using the graphic in figure 203 (µ'=103@30MHz which is 90 increased by 15%) we have:

$$L_S = N^2 A_L = 4\pi\mu' N^2 \frac{A_e}{l_e} = 11423 \text{ [nH] with } A_e \text{ in cm}^2 \text{ and } l_e \text{ in cm} \qquad \textbf{2.50}$$

Having set N=9 spires, we can find from table 1 that A_e=1.58cm^2 and l_e=14.5 cm. After deriving C_P from 2.30b we obtain:

$$C_P = \frac{1}{4\pi^2 f_B{}^2 L_S} = \frac{10^{-3}}{39{,}48 \cdot (30{,}41)^2 \cdot 11423} = 2{,}39 \text{ pF} \qquad \textbf{2.51}$$

Just for verification, let's measure the parasite capacity using the method suggested by [6]. Let's assemble a circuit like the one in figure 207 and measure the frequencies of resonance respectively with C_1=3,12pF and C_2=4,91pF (the capacitors made of silver mica I found in my drawer). We obtain respectively f_1=8,033MHz and f_2=6,982MHz. From 2.18 we can derive:

$$C_P = C_1 \frac{\frac{C_2}{C_1}\left(\frac{f_2}{f_1}\right)^2 - 1}{1 - \left(\frac{f_2}{f_1}\right)^2} = 2{,}41 pF \qquad 2.52$$

The capacities found with 2.51 and 2.52 differ one from the other by less than 1%, so I would say that the parasite capacity can be considered around 2.4pF. Now let's verify the differential mode currents.

Let's start with finding the value of the coupling factor k. We need to measure the inductances L_{pc}=0,6µH ed L_{po}=111µH with an LCR meter and using 2.9 we obtain that k=0.997; the value is close to the unit as we predicted. Then, let's build the circuit in figure 214 and measure the scatter parameter $|S_{11}|$ with the network: we are doing this because the impedance seen by the hatching line in figure 214 generates a reflection of the incident wave. Its coefficient of reflection Γ (gamma) is equal to S_{11} when the balanced port is closed on 50Ω [8], like in our case. Therefore, to have a correspondence between theory and reality in the practical measurements, we would need to obtain Z_i from the scatter parameter S_{11}. The mathematical relation between the two parameters is the following:

$$Z_i = Z_{out} \frac{1+S_{11}}{1-S_{11}} \quad 2.53$$

Where Z_i is the impedance picked up by the unbalanced port of the choke and Z_{out} is the impedance picked up by the balanced port of the choke. The return loss RL can be obtained from 2.54. This parameter expresses how much the reflected wave is reduced by the input port of the choke, when the output port is closed on Z_{out}=50Ω:

$$RL = -20Log|S_{11}| = -20Log|\Gamma| \text{ dB} \quad 2.54$$

Therefore, when we measure impedances values which are very close to Z_{out} (more or less) on the unbalanced input port, we know that the $|S_{11}|$ will be very small and the return loss very big. It's not unusual to find RL values greater than 50dB. From the S_{11} we can also calculate the SWR we have at the input port of the choke.

$$SWR = \frac{1+|S_{11}|}{1-|S_{11}|} \quad 2.55$$

The measuring setup is the one in figure 221:

Fig.221

The trace of the $|S_{11}|$ is the black one in the upper part of the graphic and it is represented in ohm over three decades (from 0.1 to 100MHz).

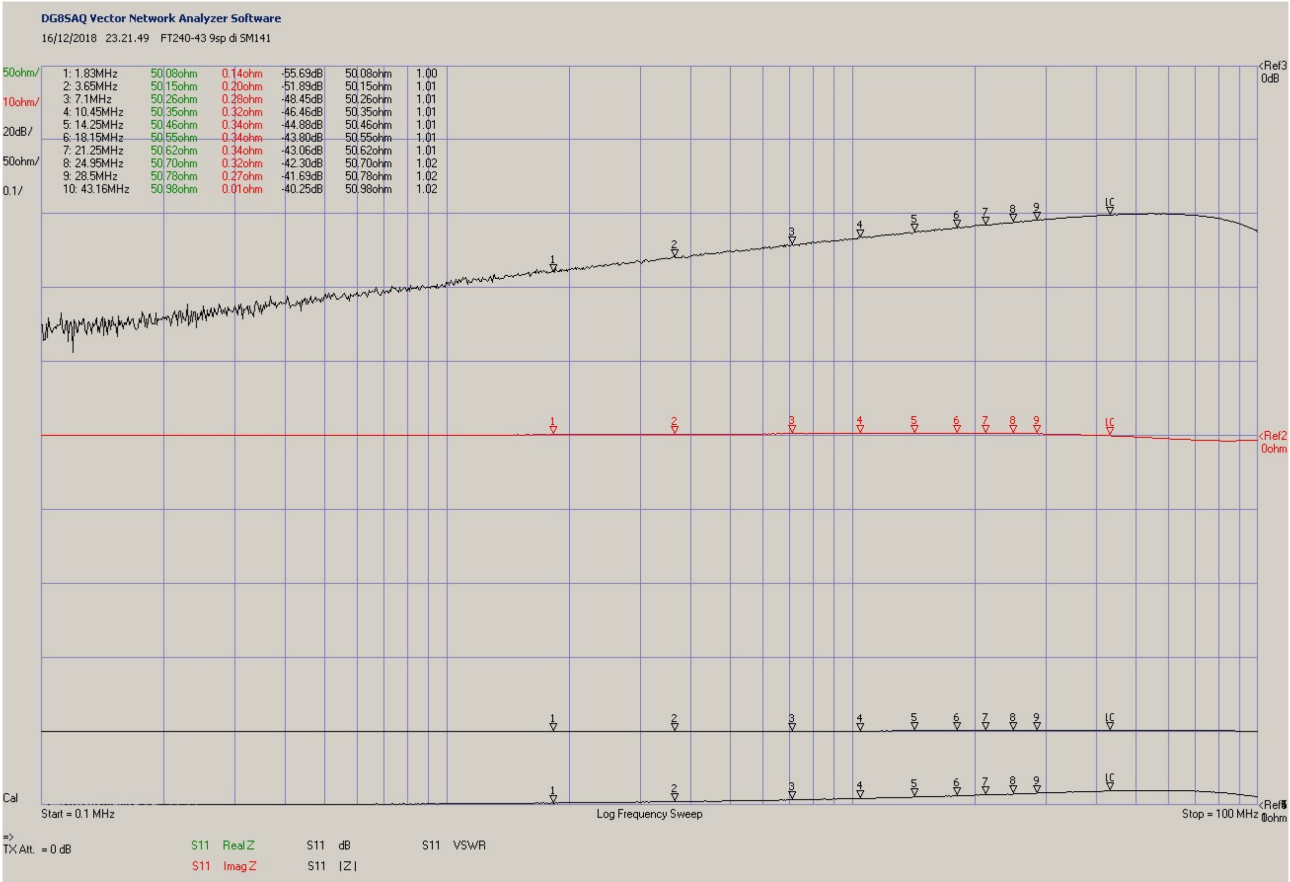

Fig.222

As you can see, figure 222 shows that the $|S_{11}|$ in dB stays always under -40dB, so Z_i always maintains close to 50 Ohm; this means that the SWR is always very close to the unit. In particular, we have:

f (MHz)	**1,830**	**3,650**	**7,100**	**10,450**	**14,225**	**18,150**	**21,250**	**24,950**	**28,500**
$|S_{11}|$ (dB)	-55,69	-51,89	-48,45	-46,46	-44,88	-43,80	-43,06	-42,30	-41,69
Re Zi (Ω)	50,08	50,15	50,26	50,35	50,46	50,55	50,62	50,70	50,78
Im Zi (Ω)	0,14	0,20	0,28	0,32	0,34	0,34	0,34	0,32	0,27
SWR	1,003	1,01	1,01	1,01	1,01	1,01	1,01	1,02	1,02

Tabella 6

As theory predicted, the presence of the choke, does not influence on the differential mode currents. For a complete picture, let's look at the values of frequency between whom the equation 2.42 is satisfied. From the graphic 223 (which represents the impedance Z_c), let's measure the values of frequency f_l=535KHz and f_h=65.1MHz in correspondence with $10 \cdot R_2 = 250\ \Omega$. The values cover the entire range of frequency of our project.

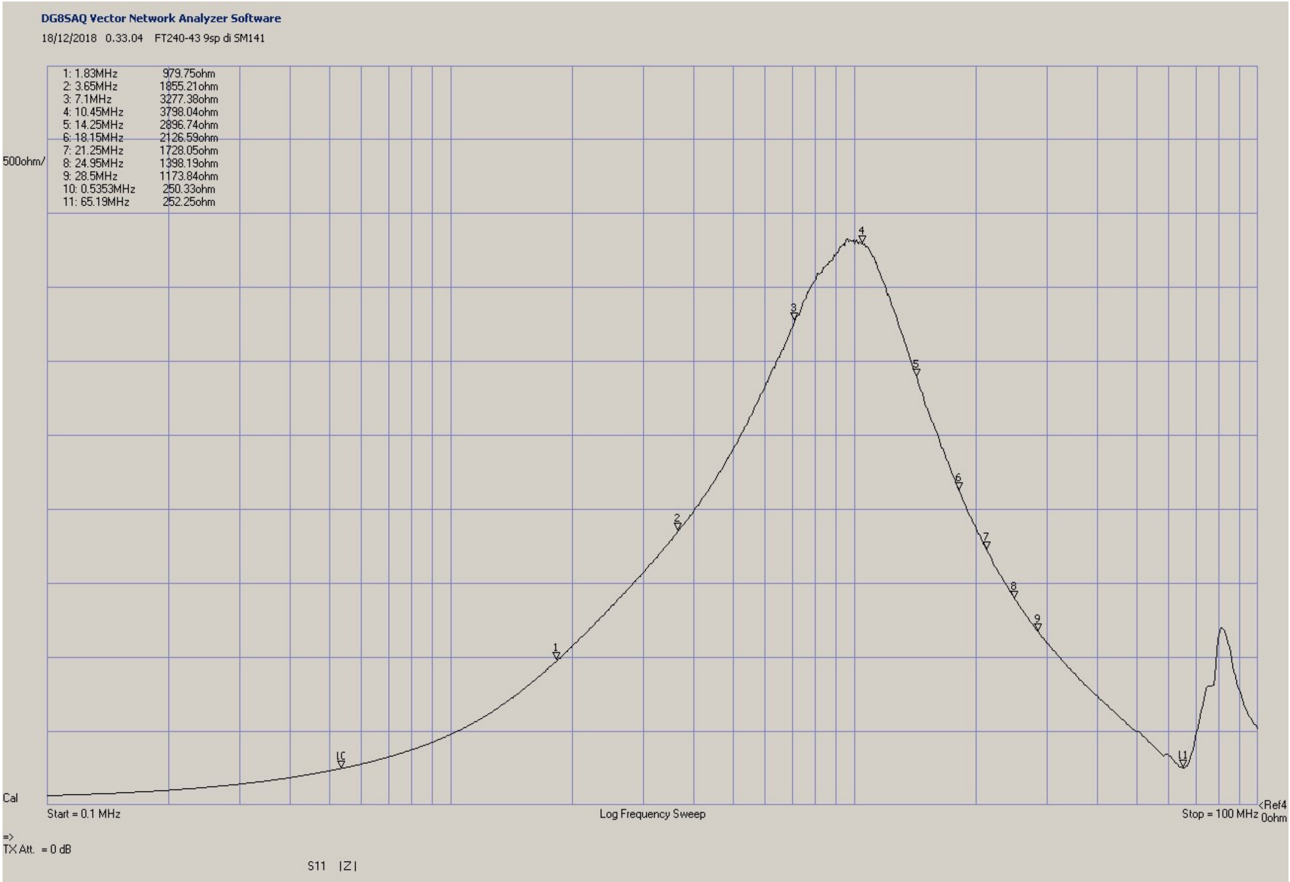

Fig.223

Final considerations

It seems like the choke we built lives up to our expectations already with 9 spires. Probably many of you would like to know what would happen if we used more spires. For this reason, I am providing table 7, with the values of the IL and point B for growing odd number of spires, until the maximum allowed with the cable SM141-50 and the ferrite FT240-43.

f (MHz)	**1,830**	**3,650**	**7,100**	**10,450**	**14,225**	**18,150**	**21,250**	**24,950**	**28,500**		**f_B (MHz)**
IL (dB) 9 spire	20,43	24,22	27,68	29,74	31,33	32,36	32,84	33,10	33,01		30,41
IL (dB) 11 spire	23,81	27,70	31,42	33,74	35,55	36,35	36,35	35,80	34,79		20,20
IL (dB) 13 spire	26,65	30,60	34,53	37,11	38,99	39,24	38,63	37,40	35,88		16,63
IL (dB) 15 spire	29,13	33,25	37,54	40,39	41,74	40,28	38,77	36,86	35,02		13,78
IL (dB) 17 spire	31,39	35,81	40,72	43,30	42,32	38,94	36,99	34,86	32,98		10,41

Table 7

The graphic in figure 224 illustrates the variations of the IL: you can notice how the measured attenuation keeps increasing in the first segment until point B (represented by a red dot). The latter moves towards lower and lower frequencies.

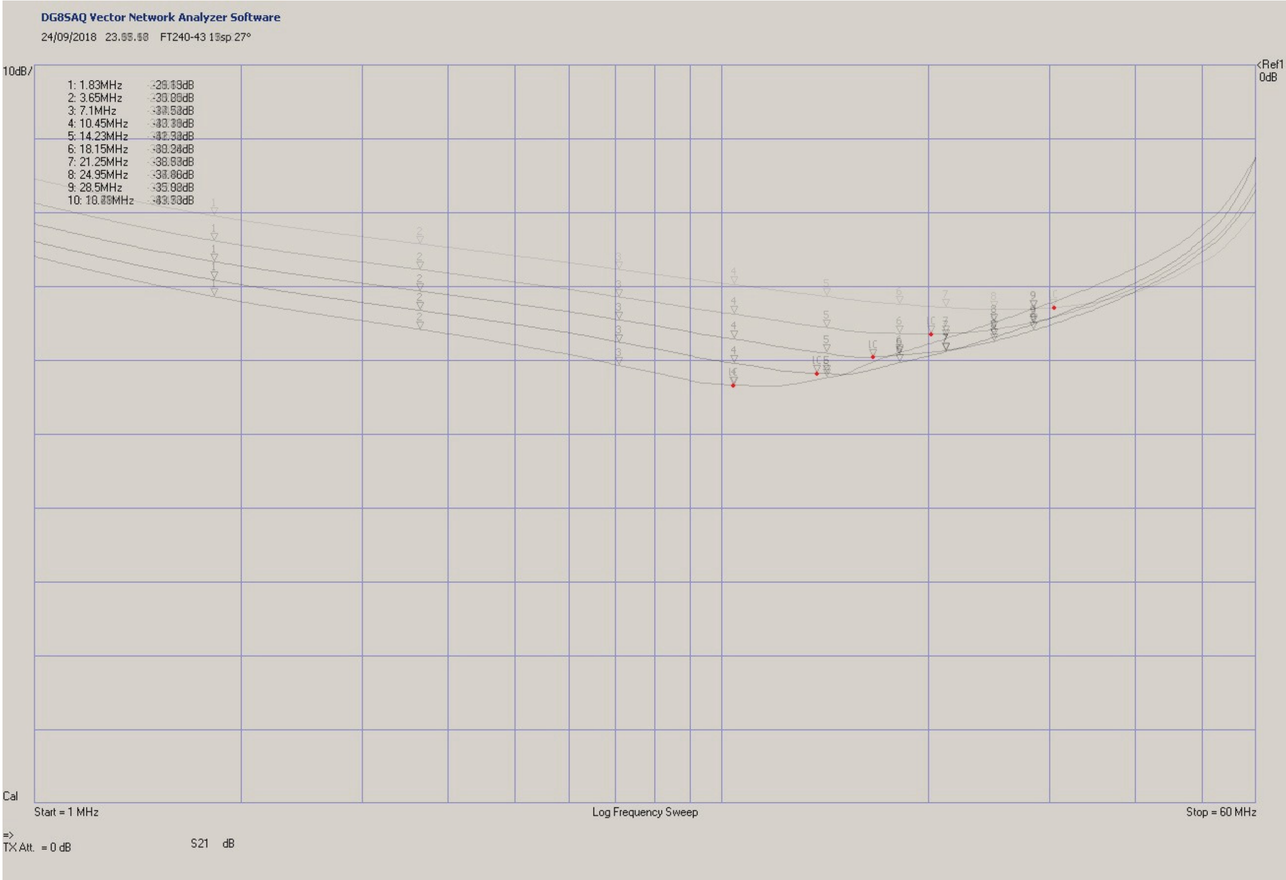

Fig.224

I also tried to wind the choke in another way (figure 225), but I did not notice any substantial variation compared to the winding in figure 219. The only difference is that the balanced and unbalanced port of the choke are on the same side, which adds a small parasite capacity due to the proximity of the two leads.

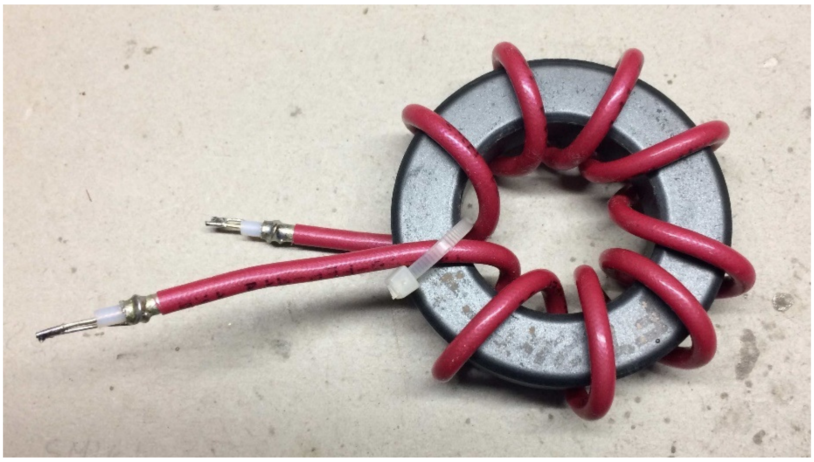

Fig.225

Out of curiosity, I built a toroid the same size of the FT240 with POM (Polyoxymethylene), commercially called Delrin®, which how the company that produces it (Dupont) named it.

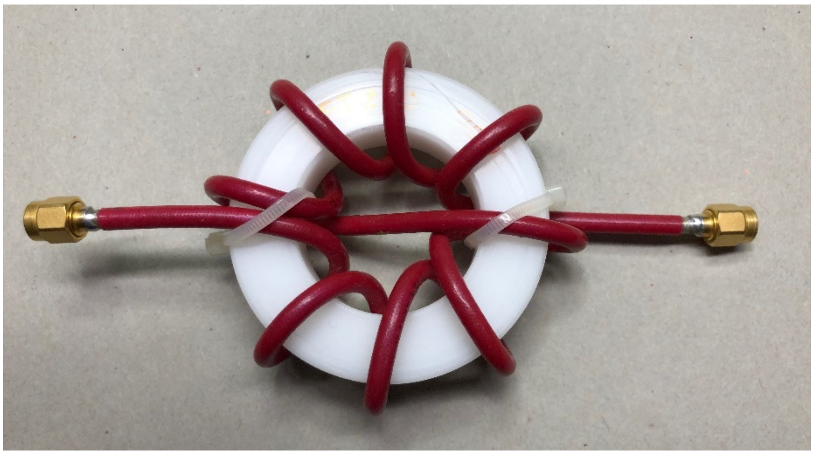

Fig.226

POM responds to radiofrequency in a similar way to PTFE (polytetrafluoroethylene), but it has way better mechanical characteristics. The magnetic permeability of POM is around the one of air, so we can consider the winding as if it was winded in air. The graphic in figure 227 shows the impedance module diagram that the choke in figure 226 offers to the common mode currents.

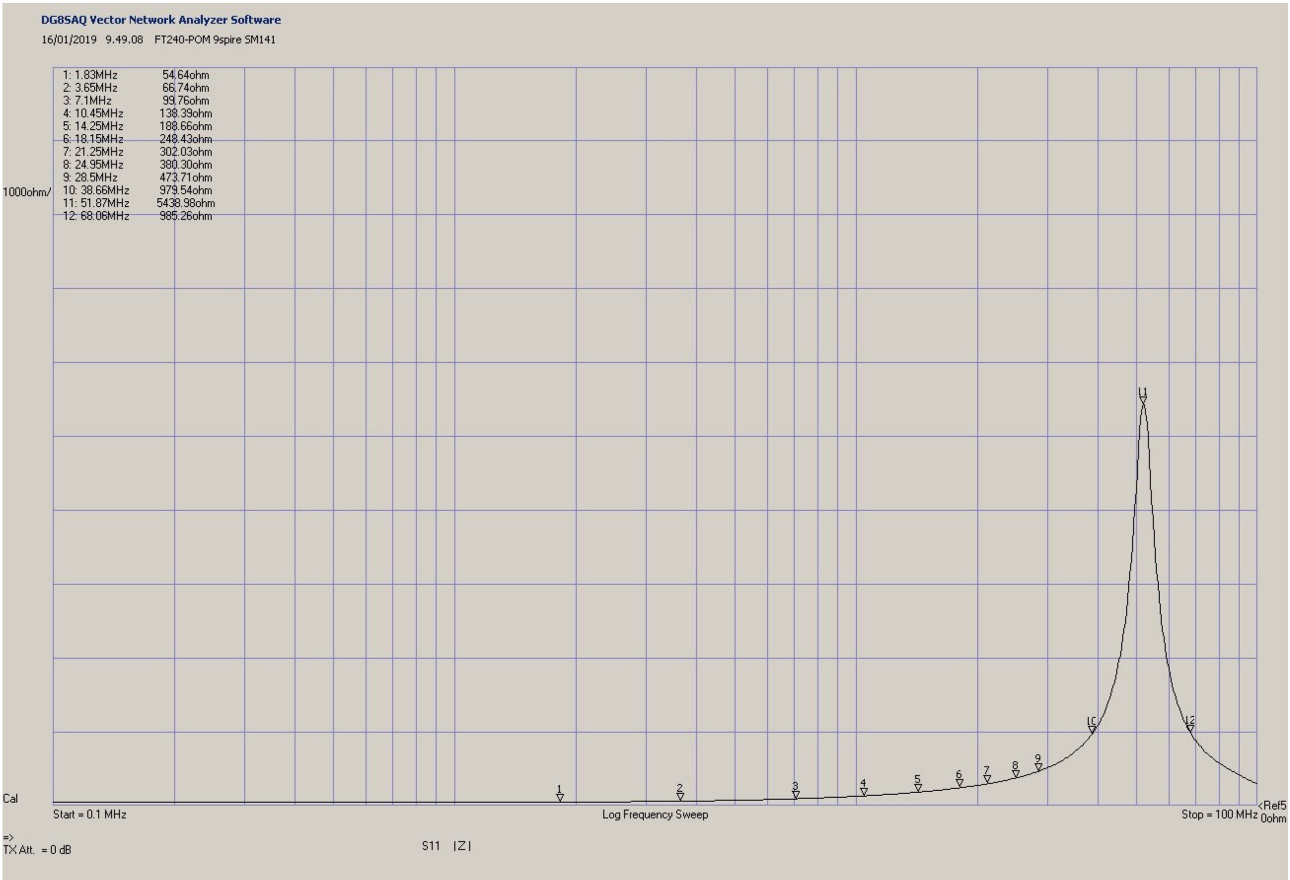

Fig.227

As you can see the auto resonance is very noticeable and shifted towards f=51.87MHz. The bandwidth B_W of the choke (at -20dB of attenuation, about 900Ω) ranges from 38.66MHz to 68.06MHz (values that correspond to about 1000Ω), and therefore it comes out being B_W=29MHz. We could think of using the choke in 6m band, but simply by getting my hand closer to it changes the shape of the trace (worsening the Q). This means that

the ferromagnetic objects in the proximity of the choke have a greater influence when the spires are winded in air, making the use of the choke itself a bitvariable.
I also winded, again, out of sheer curiosity, the 9 and the 17 spires of the deformable coaxial cable SM141-50 on a fiberglass tube having an external diameter of 16mm (I chose this specific measure not to hinder the minimum ray of curvature of the coaxial; see table 4). I then inserted into it a ferrite bar (bought from [10] with the code BF-58) having a permeability of μi=300, being 200mm long and with a diameter of 12mm. The bar has been inserted inside the fiberglass tube which had, winded on it, at first 9 spires and then 17, as shown in figure 228.

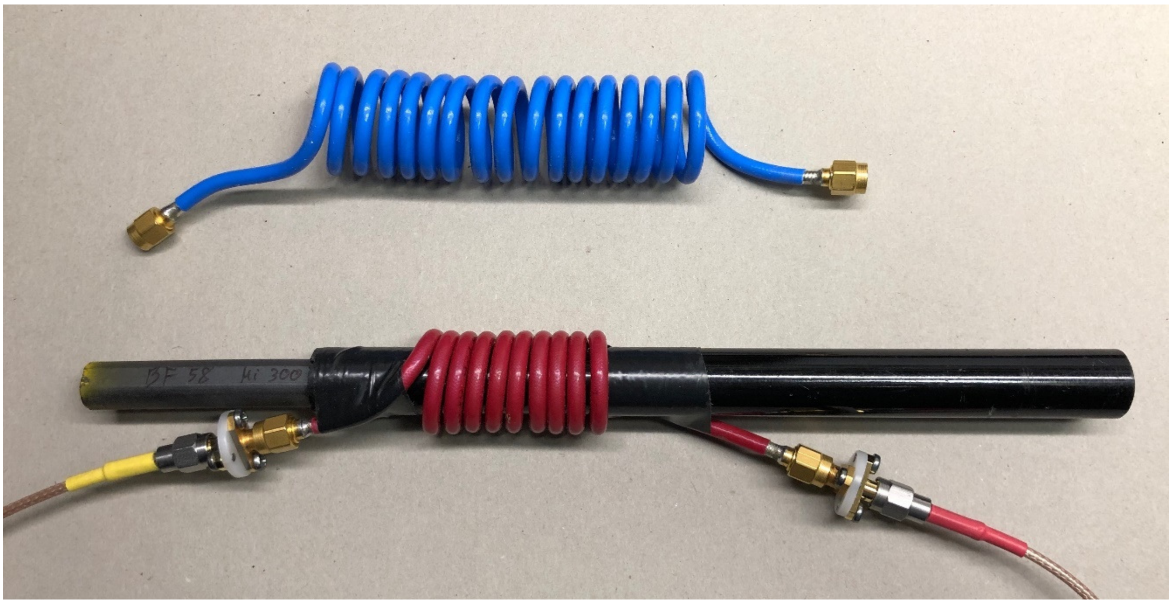

Fig.228

I then proceeded to measure the impedance of the choke as described in figure 212. The results are reported in figure 229.

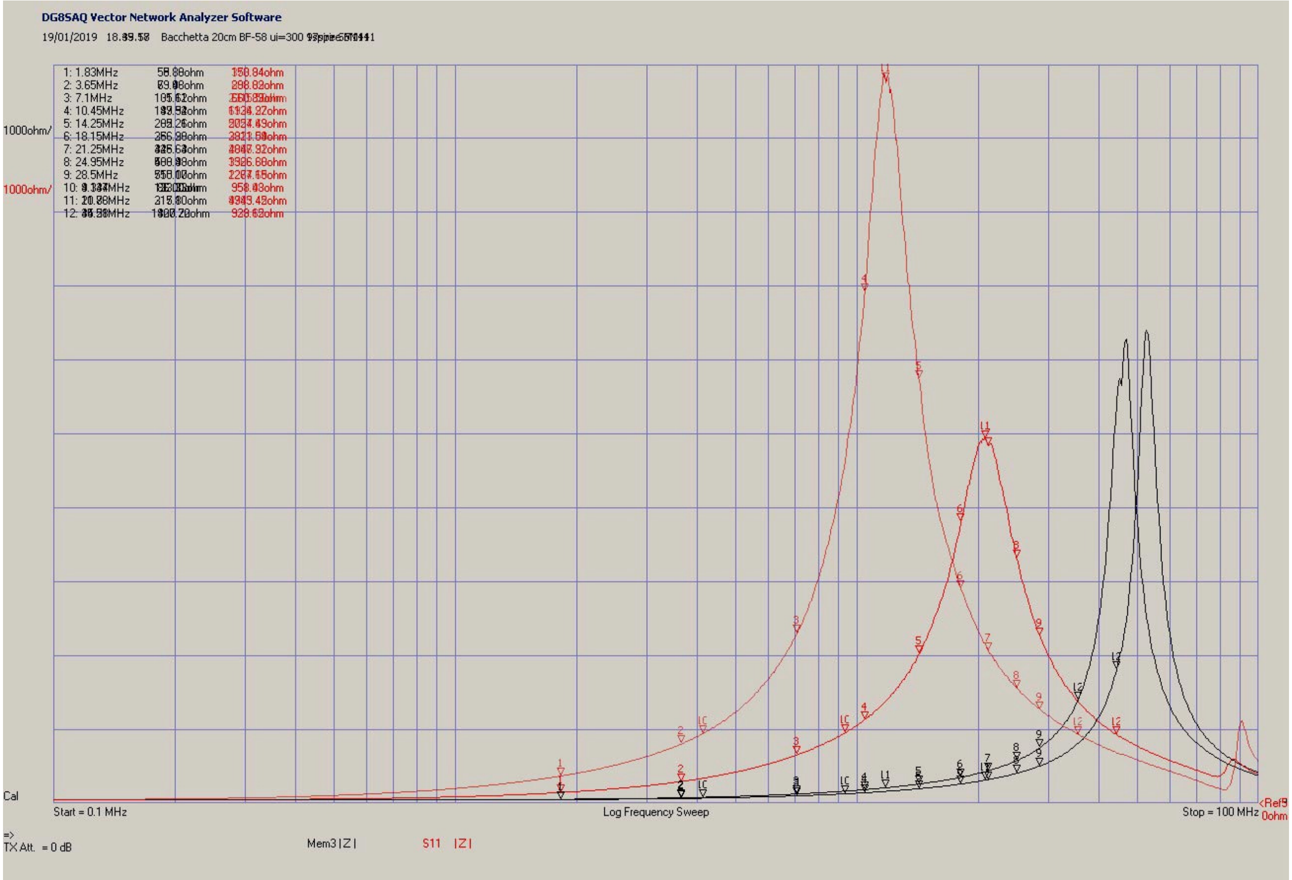

Fig.229

The black traces are the ones found without the ferrite inserted, while the red ones are the ones detected with the ferrite placed in. The taller red graphic and the lower frequency black one are the ones obtained with 17 spires, while the other two are the ones with 9. I am summing up in table 8 the values I found with the bar inserted.

9 spire	**Min**	**Picco**	**Max**
Frequenza MHz	9,34	20,86	44,28
Impedenza Ω	958	4943	929
17 spire			
Frequenza MHz	4,13	11,78	35,51
Impedenza Ω	951	9785	938

Tabella 8

It is not hard to understand that by properly choosing the number of spires it's possible to center the bands where we can reduce the CMCs. I noticed that when moving the bar more to the left or to the right, the values of frequency move to the top, while the values of the impedance decrease; this allows us, within certain limits, to better set the choke. I conducted tests even with the bar BF-55 (also bought from [10]), which has a higher permeability $\mu i=400$, but a smaller diameter of 10mm, while the length is always 200mm: this test produced very poor results. I cannot give an explanation on why this happened, because I do not know any characteristic relative to the ferrite mixture. I own, so I can only make hypothesis. I also measured the inductance factor A_L of different toroid with different gradations that I included in table 9.

	FT240-31				FT240-52				FT240-61			
FREQUENZA	A_L (nH/sp^2)	Rs (Ω)	Xs (Ω)	Zs (Ω)	A_L (nH/sp^2)	Rs (Ω)	Xs (Ω)	Zs (Ω)	A_L (nH/sp^2)	Rs (Ω)	Xs (Ω)	Zs (Ω)
1,830	1656	11,7	19,1	22,4	339	0,0	3,9	3,9	136	0,0	1,6	1,6
3,650	899	22,8	20,6	30,7	359	0,3	8,2	8,2	137	0,0	3,2	3,2
7,100	465	30,8	20,9	37,2	381	4,9	17,0	17,7	140	0,1	6,3	6,3
10,450	349	35,2	22,9	42,0	306	11,5	20,1	23,2	146	0,4	9,6	9,6
14,225	279	40,1	25,0	47,3	237	16,0	21,2	26,6	151	1,3	13,5	13,6
18,150	231	44,7	26,4	52,0	194	19,1	22,2	29,3	155	2,9	17,7	17,9
21,250	203	48,0	27,1	55,8	172	20,9	23,0	31,1	156	4,9	20,9	21,5
24,950	175	51,6	27,5	61,3	154	22,8	24,2	33,2	153	8,3	24,3	25,5
28,500	154	54,7	27,6	72,3	141	24,4	25,4	35,3	144	11,9	25,9	28,5
50,100	81	67,6	25,5	61,7	102	34,2	32,3	47,0	88	25,3	27,7	37,5

Tabella 9

In conclusion, you might wonder whether is better or not to use a choke. The answer is that it is probably better to use one where there are common mode currents that need to be blocked. The common mode currents are produced by the moving of the charges in both the two conductors of the line in the same direction; therefore, the best place to block them is a place which is the closest to where they originate. For example: if it is the case of a radiant dipole, it is better to place the choke closer to the terminals of the antenna, to avoid the cable from becoming a radiating part itself, causing a serious deformation of the radiation lobe of the antenna.

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